3.5 Sinusoidal Functions

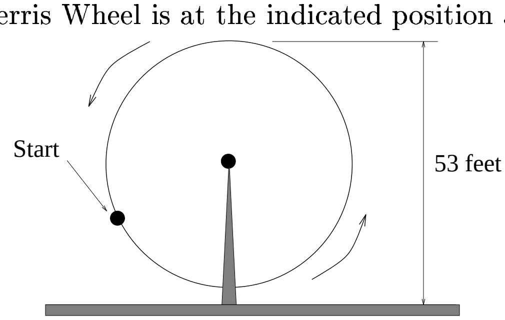
1. Find the amplitude, period, phase shift and mean of the following sinusoidal functions.
   (a) $y = \sin(x - \frac{\pi}{2})$
   (b) $y = 3\sin(x + \frac{\pi}{6})$
   (c) $y = \sin(2x - \pi) + 1$
   (d) $y = 6\sin(\pi x) - 1$
   (e) $y = -2\sin(2x - \pi) + 3$
   (f) $y = 4\sin(11x) + 11$
   (g) $y = \frac{1}{2}\sin(\frac{x}{2})$
   (h) $y = \pi\sin(-2x) - \pi$
   (i) $y = \cos(2x - \pi) + 1$
   (j) $y = \pi\cos(2x + 1) - 5$

2. Show that the functions $y = 3\sin(-3x+2)-2$, $y = 5\cos(4x)+2$ and $y = -2\cos(x+1)+1$ are sinusoidal; i.e. each can be written in the form of (4.2.1). Describe the phase shift, amplitude, mean and period of each function; sketch the graph.

3. In this problem, follow the "five step procedure" outlined following to roughly sketch the graph of the given sinusoidal function:
   (a) $y = 2\sin\left(x - \frac{\pi}{2}\right) + 2.$
   (b) $y = 2\sin\left(\frac{\pi}{4}(x + 6)\right) - 3.$
   (c) $y = 25\sin(1.3\pi + 0.4\pi t) + 28.$
   (d) $y = 8\sin(\frac{\pi}{0.6}(t - 0.3)) + 18.$
   (e) $y = 10\sin\left(\pi(x + 1)\right) + 5.$
   (f) $y = 27\sin(\frac{\pi}{183}(t - 80.5)) + 45.$
   (g) $y = 2\cos\left(\frac{\pi}{5}x - \pi\right) + 1.$

4. Marcel is anxious to learn his Chem 150 grade. He starts out standing in front of Professor Zoller's office door and paces back and forth along a straight line. He begins 20 feet in front of the office door, walks toward the door and in 4 seconds he is 2 feet from the door. He then backs off, moving back and forth sinusoidally as a function of time, all the while moving between 2 feet and 20 feet from the door. After 31 seconds, he works up the courage to knock on the door, so he walks toward the door at a constant speed of 8 ft/sec. Let $m(t)$ be the function which describes the distance from Marcel to the door after t seconds.
   (a) Give a formula for the function $m(t)$ during the first 31 seconds.
   (b) Where is Marcel located after 31 seconds?
   (c) When does he reach the door?
   (d) Give a formula for the multipart function $m(t)$, up until the time he reaches the door.
   (e) Sketch a picture of the graph of $m(t)$.

5. Your seat on a Ferris Wheel is at the indicated position at time $t = 0$.



Let t be the number of seconds elapsed after the wheel begins rotating counterclockwise. You find it takes 3 seconds to reach the top, which is 53 feet above the ground. The wheel is rotating 12 RPM and the diameter of the wheel is 50 feet. Let $d(t)$ be your height above the ground at time t.

(a) Argue that $d(t)$ is a sinusoidal function, describing the amplitude, phase shift, period and mean.

(b) When are the first and second times you are exactly 28 feet above the ground?

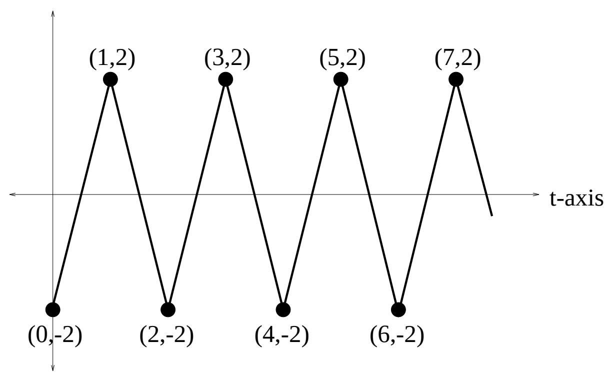
(c) After 29 seconds, how many times will you have been exactly 28 feet above the ground?

6. Suppose the high tide in Seattle occurs at 1 am and 1 pm at which time the water is 10 feet above the height of low tide. Low tides occur 6 hours after high tides. Suppose there are two high tides and two low tides every day and the height of the tide varies sinusoidally.

   (a) Find a formula for the function $y = h(t)$ that computes the height of the tide above low tide at time t. (In other words, $y = 0$ corresponds to low tide.)

   (b) What is the tide height at 11 am?

7. Digital audio technology requires that *piecewise linear functions* (functions with graphs consisting of line segments) be modeled by sinusoidal functions (or, sums of such). Let t represent the time variable in seconds. Given the *sawtooth function* $d(t)$, answer the following questions.



   (a) Find the multipart rule for $d(t)$, when $0 \leq t \leq 7.6$.

   (b) Find a formula for a sinusoidal function $s(t)$ passing through the labeled extrema.

8. A respiratory ailment called "Cheyne-Stokes Respiration" causes the volume per breath to increase and decrease in a sinusoidal manner, as a function of time. For one particular patient with this condition, a machine begins recording a plot of volume per breath versus time (in seconds). Let $b(t)$ be a function of time t that tells us the volume (in liters) of a breath that starts at time t. During the test, the smallest volume per breath is 0.6 liters and this first occurs for a breath that starts 5 seconds into the test. The largest volume per breath is 1.8 liters and this first occurs for a breath beginning 55 seconds into the test.

   (a) Find a formula for the function $b(t)$ whose graph will model the test data for this patient.

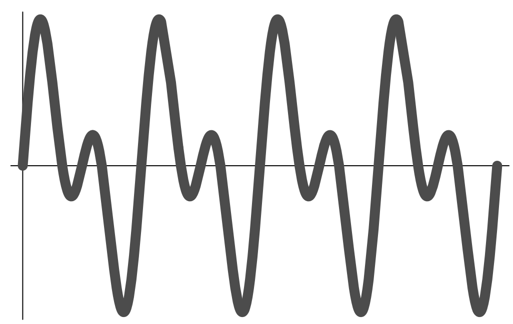
   (b) If the patient begins a breath every 5 seconds, what are the breath volumes during the first minute of the test?

9. The data below was downloaded from NASA, describing the height of the Russian Space Station MIR at 5 minute intervals.

Time	Height(km)	Time	Height(km)
1:15:00 AM	396.8	1:55:00 AM	393.1
1:20:00 AM	397.6	2:00:00 AM	391.6
1:25:00 AM	397.7	2:05:00 AM	390.1
1:30:00 AM	397.3	2:10:00 AM	388.5
1:35:00 AM	396.7	2:15:00 AM	387.6
1:40:00 AM	396.0	2:20:00 AM	387.5
1:45:00 AM	395.2	2:25:00 AM	388.3
1:50:00 AM	394.3	2:30:00 AM	390.0

Assume the height of MIR above the earth is a sinusoidal function of time and that the maximum and minimum heights both occur between 1:15 am and 2:30 am, as tabulated. Based on this maximum and minimum height data, find a sinusoidal function which models the height of MIR at time t. Discuss how the tabulated data differs from the model you have constructed.

10. Here is a graph of the function $y = \sin(x) + \sin(2x)$. Is this function sinusoidal? Give a reason.



11. Let A, B, C, D be constants with A and B positive. Show that the function $y = g(x) = A\cos(Bx - C) + D$ is a sinusoidal function by putting it in standard sinusoidal form. Find the amplitude, period, phase shift and mean.

12. A weight is attached to a spring suspended from a beam. At time $t = 0$, it is pulled down to a point 10 cm above the ground and released. After that, it bounces up and down between its minimum height of 10 cm and a maximum height of 26 cm, and its height $h(t)$ is a sinusoidal function of time t. It first reaches a maximum height 0.6 seconds after starting.

    (a) Follow the procedure outlined in this section to sketch a rough graph of $h(t)$. Draw at least two complete cycles of the oscillation, indicating where the maxima and minima occur.

    (b) What are the mean, amplitude, phase shift and period for this function?

    (c) Give four different possible values for the phase shift.

    (d) Write down a formula (as in (3.5.1)) for the function $h(t)$. Confirm it is consistent with (a) using a graphing device.

    (e) What is the height of the weight after 0.18 seconds?

    (f) During the first 10 seconds, how many times will the weight be exactly 22 cm above the floor? (Note: This problem does not require inverse trigonometry.)

13. The angle of elevation of the sun above the horizon at noon is 18^o on December 21, the 355th day of the year. The elevation is 72^o at noon on June 21, the 172nd day of the year. These elevations are the minimum and maximum elevations at noon for the year. Assume this particular year has 366 days and that the elevation on day t is given by a sinusoidal function $E(t)$.

   (a) Follow the procedure outlined in this section to sketch a rough graph of $E(t)$. Draw at least two complete cycles of the oscillation, indicating where the maxima and minima occur.

   (b) What are the mean, amplitude, phase shift and period for this function?

   (c) Give four different possible values for the phase shift.

   (d) Write down a formula (as in (3.5.1)) for the function $E(t)$. Confirm it is consistent with (a) using a graphing device.

   (e) What is the elevation at noon February 7? (January has 31 days and we're counting January 1 as the first day of the year.)

14. In Exercise 3.3.17, we studied the situation below: A bug has landed on the rim of a jelly jar and is moving around the rim. The location where the bug initially lands is described and its angular speed is given. Impose a coordinate system with the origin at the center of the circle of motion. In each of the cases, the earlier exercise found the coordinates $P(t)$ of the bug at time t. Pick three of the scenarios below and answer these two questions:

   (a) Both coordinates of $P(t)$ are sinusoidal functions in the variable t; i.e. $P(t) = (x(t), y(t))$. Put $x(t)$ and $y(t)$ in standard sinusoidal form. Find the amplitude, mean, period and phase shift for each function.

   (b) Sketch a rough graph of the functions $x(t)$ and $y(t)$ in (a) on the domain $0 \leq t \leq 9$.