3.4 Basic Properties and Trigonometric Functions

1. On your calculator you can check that $\cos(64^o) = 0.4384$ and $\sin(64^o) = 0.8988$.
   (a) Find four other angles ψ so that $\cos(\psi) = 0.4384$ and $\sin(\psi) = 0.8988$.
   (b) Find an angle ψ, $500^o \leq \psi \leq 1000^o$, so that $\cos(\psi) = 0.4384$ and $\sin(\psi) = 0.8988$.
   (c) How many angles ψ of measure between 270^o and 460^o will have $\cos(\psi) = 0.4384$?
   (d) How many angles ψ of measure between 810^o and 1000^o will have $\cos(\psi) = 0.4384$?

2. On your calculator you can check that $\cos(2.2\,rad) = -0.5885$ and $\sin(2.2\,rad) = 0.8085$.
   (a) Find four other angles ψ so that $\cos(\psi) = -0.5885$ and $\sin(\psi) = 0.8085$.
   (b) Find an angle, $10 \leq \psi\,rad \leq 18$, so that $\cos(\psi) = -0.5885$ and $\sin(\psi) = 0.8085$.
   (c) How many angles $3 \leq \psi\,rad \leq 10$ have $\cos(\psi) = -0.5885$?
   (d) How many angles $6 \leq \psi\,rad \leq 7$ have $\cos(\psi) = -0.5885$?

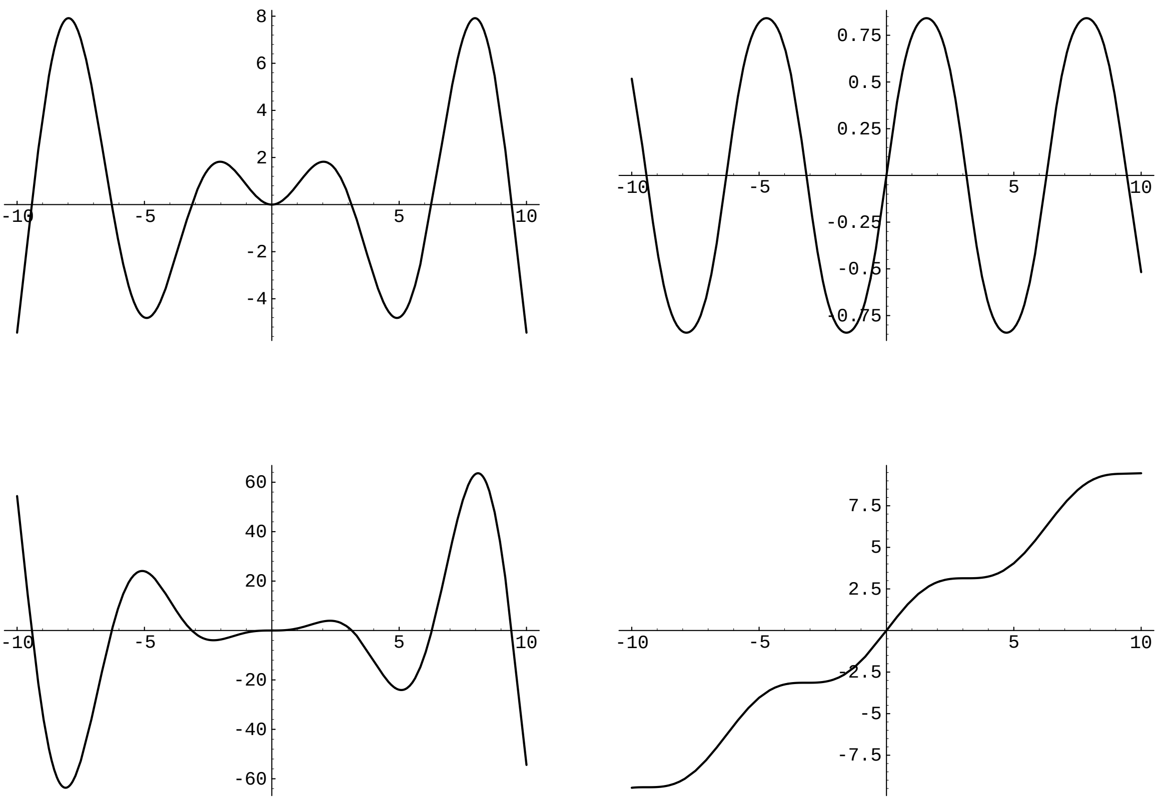
3. (a) If $\cos(\theta) = \frac{24}{25}$, what are the two possible values of $sin(\theta)$?
   (b) If $\sin(\theta) = -0.8$ and θ is in the third quadrant of the xy plane, what is $\cos(\theta)$?
   (c) If $\sin(\theta) = \frac{3}{7}$, what is $\sin(\frac{\pi}{2} - \theta)$?

4. Start with the equation $\sin(\theta) = \cos(\theta)$.
   (a) Use the unit circle interpretation of the circular functions to find the solutions of this equation; make sure to describe your reasoning.
   (b) Use the tangent function to find the solutions of this equation; make sure to describe your reasoning.
   (c) Use a graphing device to find the solutions of this equation; make sure to describe your reasoning.

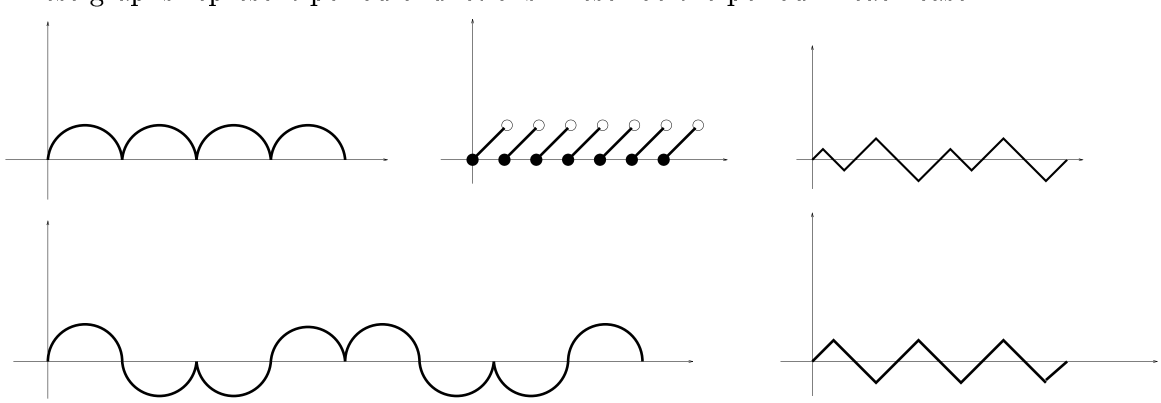
5. Consider the function $y = \frac{1}{\sin(\theta)}$. Use a graphing device to study various qualitative features (e.g., where the function is zero, undefined, positive, negative). How does the situation change if we study instead $y = \frac{1}{\sin^2(\theta)}$?

6. (a) If $f(x) = \sin(x)$ and $g(x) = 3x$, what is $f(g(x))$? What is $f(g(5))$? What is $g(f(x))$? What is $g(f(5))$?
   (b) If $f(x) = \sin(x)$ and $g(x) = x^2 + 3x - 5$, what is $f(g(x))$? What is $f(g(1))$? What is $g(f(x))$? What is $g(f(1))$?
   (c) If $f(x) = g(x) = \sin(x)$, what is $f(g(x))$? What is $f(g(1))$? Is $f(g(x)) = [\sin(x)]^2$?

7. Below are four different graphs. These are graphs of the functions: $y = x + \sin(x), y = x\sin(x), y = x^2\sin(x), y = \sin(\sin(x))$. Which function goes with each graph and why? You can use a graphical device to check yourself, but make sure you can give a reason for each choice.



8. These graphs represent periodic functions. Describe the period in each case.



9. For each situation described, which could *possibly* be described using a periodic function?

   (a) The population of rabbits in a large wooded area as a function of time.
   (b) As you pedal a bike, the height of your big toe above the ground as a function of time.
   (c) As you pedal a bike, the distance you have traveled as a function of time.
   (d) The score for the UW womens Bball team as a function of time elapsed during a game.
   (e) Your height as a function of time.
   (f) The cost of a vacant lot as a function of its size.
   (g) Your weight during the course of a single day, as a function of time.

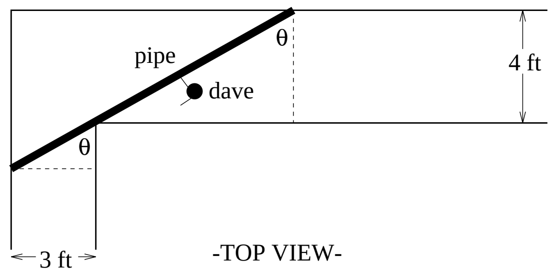
(h) The average speed of traffic on I-5 at the ship canal bridge, as a function of time.

(i) The average daily temperature in Port Angeles, WA as a function of the day of the year.

(j) The salary of a UW math professor as a function of the number of years on the faculty.

(k) Tuition at the UW as a function of time.

10.* Dave is replumbing his house and needs to carry a copper pipe around the corner of a hallway. As he cheerfully walks down the hall and rounds the corner, the pipe becomes stuck, as pictured. Assume Dave must always hold the pipe level; i.e. he can't tilt it up or down.



(a) Find a formula for the function $\ell(\theta)$ which computes the length of the longest pipe that will fit with the pictured angle θ.

(b) Describe how you could use the function in a. to find the LONGEST pipe Dave can carry around the corner.

(c) Use a graphing device to approximate the longest pipe Dave can carry around the corner.

11. You will need a graphing device for this problem. Using a single set of axes, plot these three graphs: $y = x\sin(x)$, $y = 2x\sin(x)$, $y = \frac{1}{2}x\sin(x)$.

(a) Find the intersection points (nodes) where the three graphs simultaneously intersect.

(b) Describe the appearence of the graph of $y = cx\sin(x)$ for various values of c.

12. Using (3.4.4) and (3.4.5), verify that we have this trigonometric identity:

$$\cos(\theta) = \sin(\frac{\pi}{2} + \theta).$$

Use this identity to rewrite each function $y = \cos(3)$, $y = \cos(3x)$ and $y = \cos(3x + 1)$ as a function of x which involves the sine function.