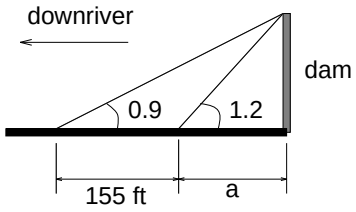
3.3 The Circular Functions

1. (a) Using the circular functions, compute the distance d in Example 3.1.6 in §3.1; compare this to the arc length s.

   (b) Go back to Chord Approximation in 3.1.7 and give a formula for the EXACT length of the chord in terms of the arc length s.

   (c) Return to Exercise 3.1.8 in §3.1 and compute the EXACT diameter of the moon.

2. The top of the *Boulder Dam* has an angle of elevation of 1.2 radians from a point on the Colorado River. Measuring the angle of elevation to the top of the dam from a point 155 feet farther down river is 0.9 radians; assume the two angle measurements are taken at the same elevation above sea level. How high is the dam?



3. A machinist needs to drill 5 holes in a circular plate. The centers of the holes must be 12 inches from the center of the plate and spaced equidistant from each other. How far apart (straight line distance) will the centers of the holes be placed? What is the arc length between the hole centers?

4. The *Leaning Tower of Pisa* was originally 179 feet high. In 1990 it was tilted at an angle of 85^o above horizontal.

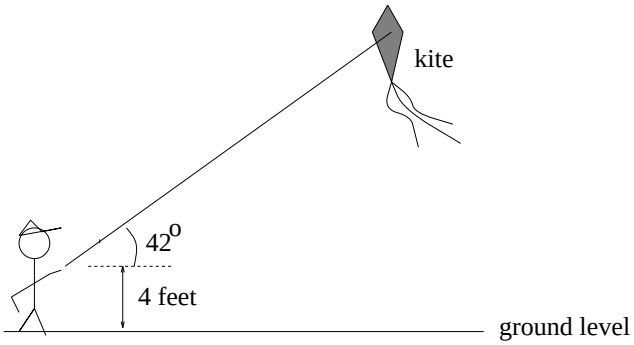
   (a) Find the length of its shadow when the sun is directly overhead.

   (b) If the angle which the tower is tilted is decreasing at the rate of $0^o07'15''$ per year, what will be the length of the shadow when the sun is directly overhead in the year 2020?

5. A kite is attached to 300 feet of string, which makes a 42 degree angle with the level ground. The kite pilot is holding the string 4 feet above the ground.

   (a) How high above the ground is the kite?

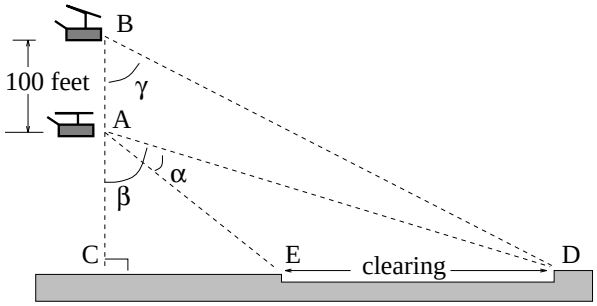
   (b) Suppose that power lines are located 250 feet in front of the kite flyer. Is any portion of the kite or string over the power lines?



6. A radio station obtains a permit to increase the height of their radio tower on Queen Anne Hill by no more than 100 feet. You are the head of the Queen Anne Community Group and one of your members asks you to make sure that the radio station does not exceed the limits of the permit. After finding a relatively flat area nearby the tower (not necessarily the same altitude as the bottom of the tower), and standing some unknown distance away from the tower, you make three measurements all at the same height above sea level. You observe that the top of the old tower makes an angle of 39 degrees above level. You move 110 feet away from the original measurement and observe that the old top of the tower now makes an angle of 34 degrees above level. Finally, after the new construction is complete, you observe that

the new top of the tower, from the same point as the second measurement was made, makes an angle of 40 degrees above the horizontal. All three measurements are made at the same height above sea level and are in line with the tower. Find the height of the addition to the tower.

7. The crew of a helicopter needs to land temporarily in a forest and spot a flat horizontal piece of ground (a clearing in the forest) as a potential landing site, but are uncertain whether it is wide enough. They make two measurements from A (see picture) finding $\alpha = 25^o$ and $\beta = 54^o$. They rise vertically 100 feet to B and measure $\gamma = 47^o$. Determine the width of the clearing to the nearest foot.



8. A merry-go-round is rotating at the constant angular speed of 3 RPM counterclockwise. The platform of this ride is a circular disc of radius 24 feet. You jump onto the ride at the location pictured below.

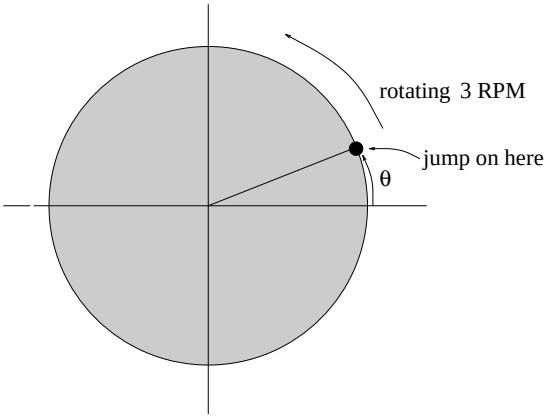
   (a) If $\theta = 34^o$, then what are your xy-coordinates after 4 minutes?

   (b) If $\theta = 20^o$, then what are your xy-coordinates after 45 minutes?

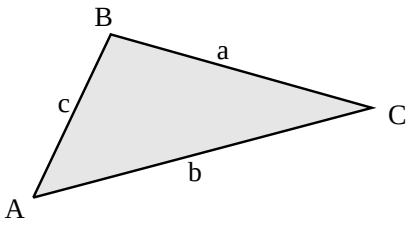
   (c) If $\theta = -14^o$, then what are your xy-coordinates after 6 seconds? Draw an accurate picture of the situation.

   (d) If $\theta = -2.1$ rad, then what are your xy-coordinates after 2 hours and 7 seconds? Draw an accurate picture of the situation.

   (e) If $\theta = 2.1$ rad, then what are your xy-coordinates after 5 seconds? Draw an accurate picture of the situation.



9. Consider the triangle $\triangle ABC$ pictured, with vertices A, B, C and sides a, b, c. Show that $\text{Area}(\triangle ABC) = \frac{1}{2}bc\sin(\angle BAC) = \frac{1}{2}ac\sin(\angle ABC) = \frac{1}{2}ab\sin(\angle BCA)$.



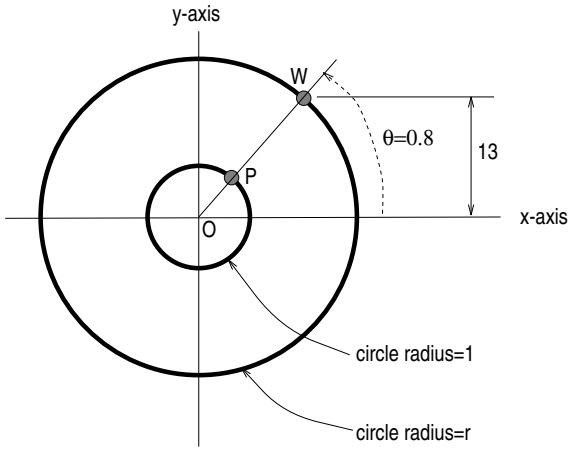
10. Go back to Example 3.3.7 and answer these additional questions:

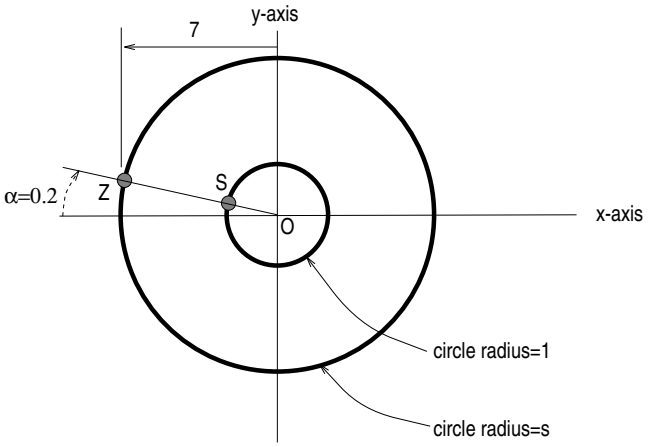
    (a) When will Michael (resp. Angela) first enter the second (resp. fourth) quadrant?

    (b) What are the linear speeds of Michael and Angela in "meters/sec"?

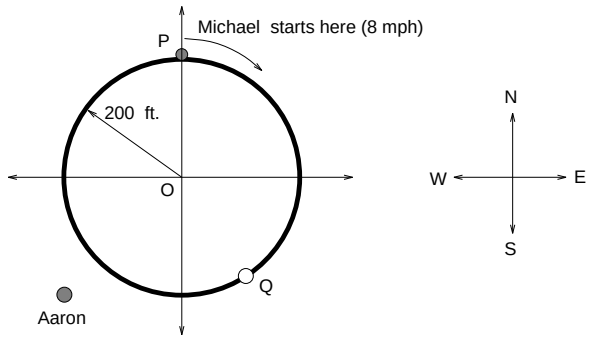
(c) Who eventually passes who? When and where does this happen?

11. In the left-hand picture, find a circle of radius r so that the vertical coordinate of the point W is 13. In the right-hand picture, find a circle of radius s so that the horizontal coordinate of Z is -7.





12.* Michael is running 8 mph clockwise around a circular track of radius 200 feet. Michael begins at the Northernmost point on the track. Aaron is located 200 feet West and 200 feet South of the center of the circular track.



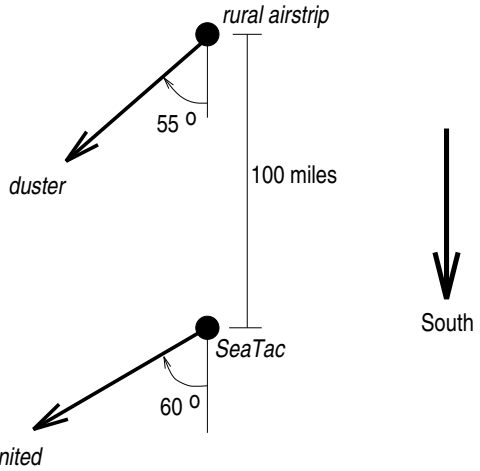
(a) If Q is Michael's location after 45 seconds, what are the coordinates of Q?

(b) What is the distance from Aaron's location to Q?

(c) Aaron starts running toward Q (constant speed in a straight line) at the instant Michael starts running toward Q. Aaron plans to tackle Michael the instant he arrives at Q. How fast should Aaron run?

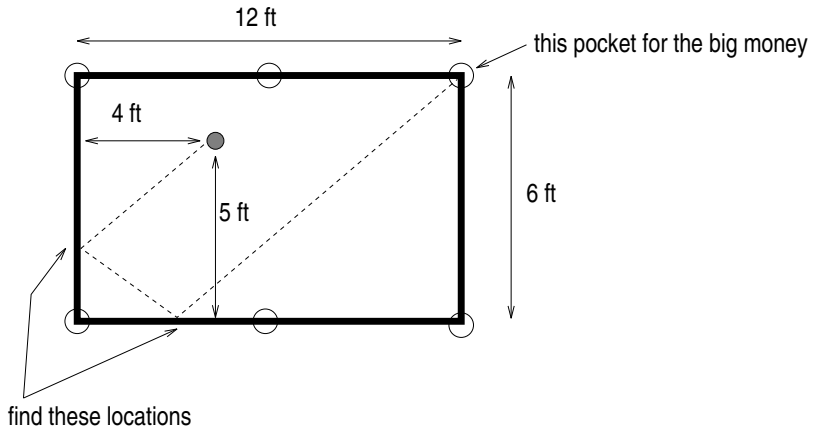
(d) As in c., where is Michael located when Aaron first crosses the circular track?

13. A United flight departs SeaTac airport heading 60^o clockwise from South. A crop duster leaves a rural airstrip located 100 miles due North of SeaTac and is heading 55^o clockwise from South. Where will the two lines of flight cross?



14. You are defending your title as the *US Billiard Champion*. Your final shot requires playing off the left and bottom cushions into the top right corner pocket, as indicated by the dotted

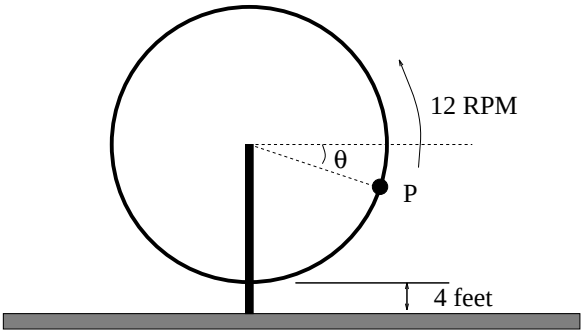
path. For the big money, where should you aim to hit the left cushion and where will the ball strike the bottom cushion? (We will return to this problem in Exercise 3.6.12 and determine the angles involved.)



15. (a) Find the equation of a line passing through the point (-1,2) and making an angle of 13^o with the x-axis. (Note: There are two answers; find them both.)

    (b) Find the equation of a line making an angle of 8^o with the y-axis and passing through the point (1,1). (Note: There are two answers; find them both.)

16. John has been hired to design an exciting carnival ride. Tiff, the carnival owner, has decided to create the worlds greatest ferris wheel. Tiff isn't into math; she simply has a vision and has told John these constraints on her dream: (i) the wheel should rotate counterclockwise with an angular speed of 12 RPM; (ii) the linear speed of a rider should be 200 mph; (iii) the lowest point on the ride should be 4 feet above the level ground. Recall, we worked on this in Exercise 3.2.14.



    (a) Impose a coordinate system and find the coordinates $T(t) = (x(t), y(t))$ of Tiff at time t seconds after she starts the ride?

    (b) Find Tiff's coordinates the instant she becomes a human missile.

    (c) Find the equation of the tangential line along which Tiff travels the instant she becomes a human missile. Sketch a picture indicating this line and her initial direction of motion along it when the seat detaches.

17. In the pictures below, a bug has landed on the rim of a jelly jar and is moving around the rim. The location where the bug initially lands is described and its angular speed is given. Impose a coordinate system with the origin at the center of the circle of motion. In each of the cases, answer these questions:

    (a) Find an angle θ_o in standard central position that gives the bugs initial location. (In some cases, this is the angle given in the picture; in other cases, you will need to do something.)

    (b) The location angle of the bug at time t is given by the formula $\theta(t) = \theta_o + \omega t$. Plug in the values for θ_o and ω to explicitly obtain a formula for $\theta(t)$.

    (c) Find the coordinates of the bug at time t.

    (d) What are the coordinates of the bug after 1 second? After 0 seconds? After 3 seconds? After 22 seconds?