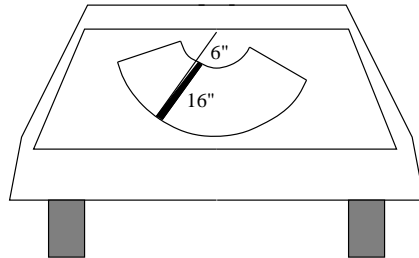


3.1 Measuring an Angle

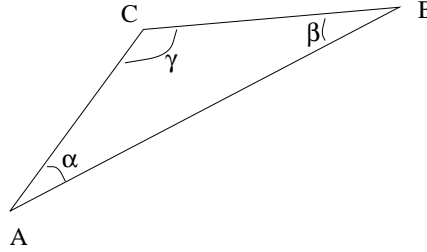
- Let $\angle AOB$ be an angle of measure θ .
 - Convert $\theta = 13.4^\circ$ into degrees/minutes/seconds and into radians.
 - Convert $\theta = 1^\circ 4' 44''$ into degrees and radians.
 - Convert $\theta = 0.1$ radian into degrees and degrees/minutes/seconds.
 - Convert $\theta = 1'$ into radians.
 - Convert $\theta = 1''$ into radians.
- Convert the given angle from radian to degree measure, then sketch a picture of the corresponding standard central position angle inside the unit circle: $1, 11, 111, \frac{\pi}{7}, \frac{-11\pi}{5}$.
 - Convert the given angle from degree to radian measure, then sketch a picture of the corresponding standard central position angle inside the unit circle: $150^\circ, 18^\circ, -40^\circ, 311^\circ, \sqrt{195}^\circ$.
- A water treatment facility operates by dripping water from a 60 foot long arm whose end is mounted to a central pivot. The water then filters through a layer of charcoal. The arm rotates once every 8 minutes.
 - Find the area of charcoal covered with water after 1 minute.
 - Find the area of charcoal covered with water after 1 second.
 - How long would it take to cover 100 square feet of charcoal with water?
 - How long would it take to cover 3245 square feet of charcoal with water?
- The rear window wiper blade on a station wagon has a length of 16 inches. The wiper blade is mounted on a 22 inch arm, 6 inches from the pivot point.



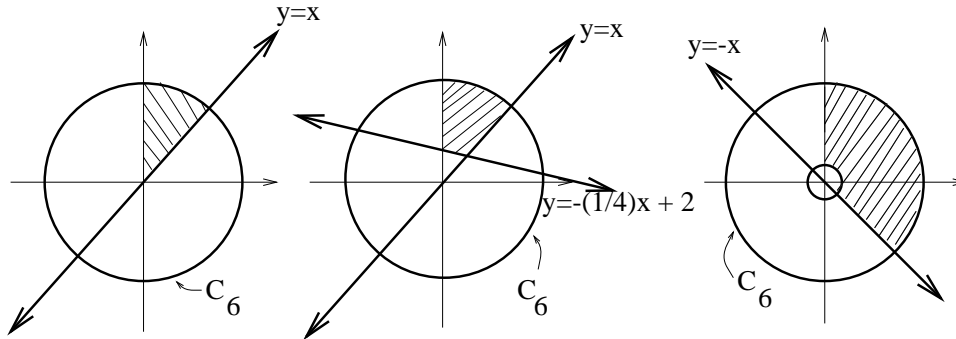
- If the wiper turns through an angle of 110° , how much area is swept clean?
- Through how much of an angle would the wiper sweep if the area cleaned was 10 square inches?
- Suppose bug A lands on the end of the blade farthest from the pivot. Assume the wiper turns through an angle of 110° . In one cycle (back and forth) of the wiper blade, how far has the bug traveled?
- Suppose bug B lands on the end of the wiper blade closest to the pivot. Assume the wiper turns through an angle of 110° . In one cycle of the wiper blade, how far has the bug traveled?
- Suppose bug C lands on an intermediate location of the wiper blade. Assume the wiper turns through an angle of 110° . If bug C travels 28 inches after one cycle of the wiper blade, determine the location of bug C on the wiper blade.

5. Suppose a triangle $\triangle ABC$ has the given two angles α, β , find the third angle γ and draw a picture for each situation.

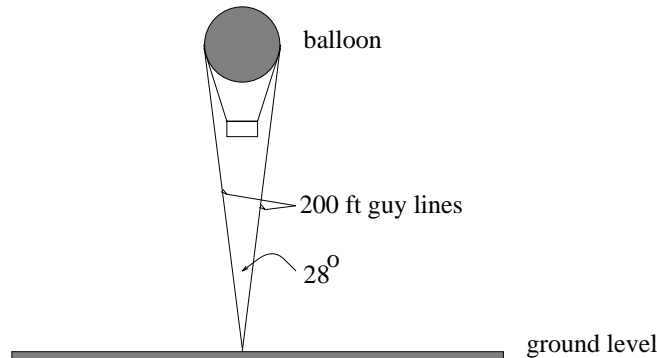
- (a) $\alpha = 30^\circ 4'$, $\beta = (44.6)^\circ$
 (b) $\alpha = 130^\circ 40' 18''$, $\beta = 0.3$ radians
 (c) $\alpha = 1'$, $\beta = \frac{\pi}{11}$ radians



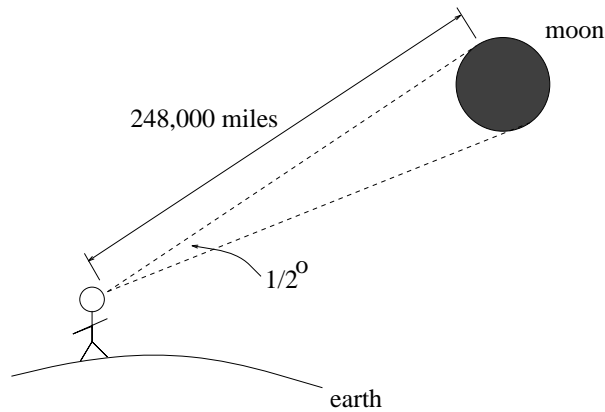
- 6.* Let C_6 be the circle of radius 6 centered at the origin in the xy -coordinate system. Compute the areas of the shaded regions in the picture below; the inner circle in the rightmost picture is the unit circle:



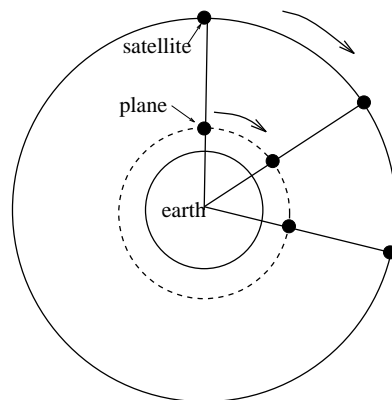
7. Suppose you are standing directly below a balloon holding guy lines which are known to be 200 feet long and fastened to the diameter of the balloon. As you stare up, you notice the angle indicated is 28° . Estimate the diameter of the balloon using (3.1.7).



8. Astronomical measurements are often made by computing the small angle formed by the extremities of a distant object and using the estimating technique in (3.1.7). In the picture below, the full moon is shown to form an angle of $\frac{1}{2}^\circ$ when the distance indicated is 248,000 miles. Estimate the diameter of the moon.



9. Suppose two locations on the earth have the same longitude and their latitudes differ by θ° . Assume the radius of the earth is 3960 miles. What is the shortest distance between the two locations if $\theta = 1''$?
10. A *nautical mile* is a unit of distance frequently used in ocean navigation. It is defined as the length of an arc s along a great circle on the earth when the subtending angle has measure $1' =$ one minute. Assume the radius of the earth is 3960 miles.
 - (a) Find the length of one nautical mile to the nearest 10 feet.
 - (b) A vessel which travels one nautical mile in one hours time is said to have the speed of one *knot*; this is the usual navigational measure of speed. If a vessel is traveling 26 knots, what is the speed in mph (miles per hour)?
 - (c) If a vessel is traveling 18 mph, what is the speed in knots?
11. The location of Vancouver, B.C. (Canada) is $123^\circ 7' W$, $49^\circ 16' North$. What is the shortest distance from Seattle to Vancouver? From Vancouver to the Equator? From Seattle to the South Pole?
12. A aircraft is flying at the speed of 500 mph at an elevation of 10 miles above the earth, beginning at the North pole and heading South along the Greenwich meridian. A spy satellite is orbiting the earth at an elevation of 4800 miles above the earth in a circular orbit in the same plane as the Greenwich meridian. Miraculously, the plane and satellite always lie on the same radial line from the center of the earth. Assume the radius of the earth is 3960 miles.



- (a) When is the plane directly over a location with latitude $74^\circ 30' 18'' N$?
- (b) How fast is the satellite moving?
- (c) When is the plane directly over the equator?

- (d) How far has the plane traveled (beginning over the North pole) when it is directly over the equator?
- (e) How far has the satellite traveled (beginning over the North pole) when it is directly over the equator?
13. During aerial spraying of insecticide to combat the Gypsy moth, a helicopter crew is assigned a sector of forest with central angle of $\frac{3}{7\pi^2}$ radians and a radius of 3.8 miles. How many square miles of forest will the crew spray?
14. A 4 ft. long pendulum swings back and forth along a 3 ft. long arc. Sketch a picture and find the number of degrees through which the pendulum passes in one swing.
15. Find the area of the sector of a circle of radius 11 inches if the measure θ of a central angle of this sector is:
- (a) $\theta = 45^\circ$
- (b) $\theta = 80^\circ$
- (c) $\theta = 3$ radians
- (d) $\theta = 2.46$ radians
- (e) $\theta = 35^\circ 24' 2''$
16. Divide a circle of radius r into 400 equal sized wedges, beginning with the point $(r, 0)$ where the circle crosses the x -axis. The interior angle of each equally sized wedge is defined to have measure equal to 1 *grad*.
- (a) Find the length of the arc subtended by the given angle inside a circle of radius 1 (the unit circle): 1 grad, 80 grad, 100 grad, 200 grad, 248 grad, 388 grad, θ grad.
- (b) How do the answers in a. change if we instead use a circle of radius r ?
- (c) Determine the order of increasing size for each given triplet of angles:
- i. 1 grad, 1 rad, 1° .
- ii. 60 grad, 3.2 rad, 200° .
- iii. 40 grad, 1 rad, 30° .
- iv. 64 grad, 1 rad, 57° .
- v. 160 grad, $\frac{2\pi}{3}$ rad, 115° .