2.9 Rational Functions

1. Give the domain of each of the following functions. Find the zeros. Sketch a graph and indicate any vertical or horizontal asymptotes. Give equations for the asymptotes.

   (a) $f(x) = \frac{2x}{x-1}$ (b) $g(x) = \frac{3x+2}{2x-5}$ (c) $h(x) = \frac{x^2-x-6}{x+2}$

   (d) $p(x) = \frac{x^2+x+1}{x-5}$ (e) $h(x) = \frac{x-3}{x^2-3x-4}$ (f) $f(x) = \frac{3x^2+6x+3}{x^2+2x}$

   (g) $g(x) = \frac{2x^2-7x+3}{x^2-x-2}$ (h) $q(x) = \frac{5}{x^2+2x+3}$

2. In 1975 I bought an old Martin ukulele for \$200. In 1995 a similar uke was selling for \$900. In 1980 I bought a new Kamaka uke for \$100. In 1990 I sold it for \$400.
   (a) Give a linear model relating the price p of the Martin uke to the year t. Take $t = 0$ in 1975.
   (b) Give a linear model relating the price q of the Kamaka uke to the year t. Again take $t = 0$ in 1975.
   (c) When is the value of the Martin twice the value of the Kamaka?
   (d) Give a function $f(t)$ which gives the ratio of the price of the Martin to the price of the Kamaka.
   (e) In the long run, what will be the ratio of the prices of the ukuleles?

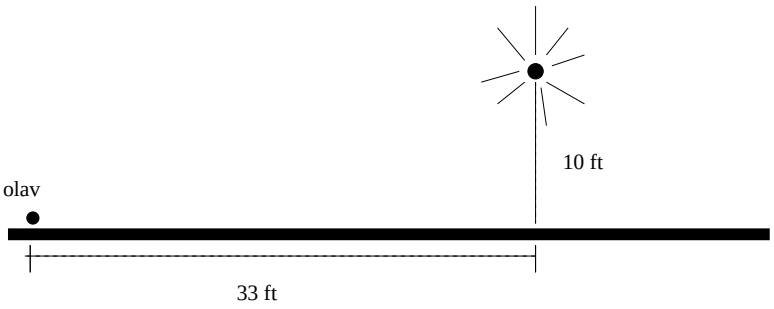
3. Isobel is producing and selling casette tapes of her rock band. When she had sold 10 tapes, her net profit was \$6. When she had sold 20 tapes, however, her net profit had shrunk to \$4 due to increased production expenses. But when she had sold 30 tapes, her net profit had rebounded to \$8.
   (a) Give a quadratic model relating Isobel's net profit y to the number of tapes sold x.
   (b) Divide the profit function in part (a) by the number of tapes sold x to get a model relating average profit w per tape to the number of tapes sold.
   (c) How many tapes must she sell in order to make \$1.20 per tape in net profit?

4. Clovis decides to go into the wholesale electricity business and buys a coal fired power plant. Unfortunately if he spends no money on pollution control, the plant will spew 10 tons of unburnt hydrocarbons into the air each year. If he spends \$10,000, he can cut this down to 5 tons a year. But no matter how much he is willing to spend, he cannot cut the pollution to below one half a ton per year.
   (a) Give a linear-to-linear rational model relating tons of unburnt hydrocarbons y to thousands of dollars spent x.
   (b) If he spends \$12,000 on pollution control, how many tons of hydrocarbons will he spew in a year?
   (c) How much must he spend to reduce pollution by 92%?

5. For each of the following, find a formula for $f^{-1}(x)$. Graph $y = f^{-1}(x)$ and give its domain.
   (a) $f(x) = \frac{2x+3}{5x+7}$ (b) $f(x) = \frac{3x+5}{7x-3}$

6. A street light is 10 feet North of a straight bike path that runs East-West. Olav is bicycling down the path at a rate of 15 MPH. At noon, Olav is 33 feet West of the point on the bike path closest to the street light. (See the picture). The relationship between the intensity C of light (in candlepower) and the distance d (in feet) from the light source is given by $C = \frac{k}{d^2}$, where k is a constant depending on the light source.
   (a) From 20 feet away, the street light has an intensity of 1 candle. What is k?

(b) Find a function which gives the intensity of the light shining on Olav as a function on time, in seconds.
(c) When will the light on Olav have maximum intensity?
(d) When will the intensity of the light be 2 candles?



7. Oscar is hunting magnetic fields with his gauss-meter. As he goes down the hall the meter stays fixed at a low level of 0.2. As he reaches the door at the end, the meter starts to rise. When he has gone 6 feet into the next room, the meter reads 2.3. Eight feet into the room, the meter reads 4.4.
(a) Give a linear-to-linear rational model relating the meter reading y to how many feet x Oscar has gone into the room.
(b) How far must he go for the meter to reach 10? 100?
(c) How far into the room is the source of the field?

8. (a) Find a rational function $y = f(x)$ whose graph has vertical asymptotes $x = 2$ and $x = -3$, and only crosses the x-axis at the point $(1, 0)$. (b) Find one such that $f(7) = 3$. (c) Can you find another function like the one in (b)?

9. The number of customers in a local dive shop depends on the amount of money spent on advertising. If the shop spends nothing on advertising, there will be 100 customers/day. If the shop spends \$100, there will be 200 customers/day. As the amount spent on advertising increases, the number of customers/day increases and approaches (but never exceeds) 400 customers/day.

   (a) Find a linear to linear rational function $y = f(x)$ that calculates the number y of customers/day if $\$x$ is spent on advertising.
   (b) How much must the shop spend on advertising to have 300 customers/day.
   (c) Assume the shop spends enough on advertising to have 300 customers/day (i.e. the amount you calculated in (b)). How many customers will the dive shop have in a month? (Assume 30 days in a month).
   (d) Sketch the graph of the function $y = f(x)$ on the domain $0 \leq x \leq 5000$.
   (e) Find the rule, domain and range for the inverse function from part (d). Explain in words what the inverse function calculates.