## 2.6 Arithmetic

1. In each of the following cases, find a formula for $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$ and $(\frac{f}{g})(x)$. In the division case, comment on any conditions required for the domain values.

   (a) $f(x) = 2x - 4$ and $g(x) = 18x^2 + 3x$.

   (b) $f(x) = -24$ and $g(x) = 8x^2 + 7$.

   (c) $f(x) = 2 - \sqrt{5-(x-3)^2}$ and $g(x) = x + 1$.

   (d) $f(x) = 21x^2 + 2x - 1$ and $g(x) = 8x^2 - 3x - 2$.

   (e) $f(x) = 0$ and $g(x) = x^3 + x$.

   (f) $f(x) = \frac{x}{x^2+1}$ and $g(x) = \frac{1}{1+\frac{1}{x}}$.

2. (a) Suppose $y = \frac{x^4}{x^2-3}$. Find simpler functions $f(x)$ and $g(x)$ so that $y = \frac{f(x)}{g(x)}$.

   (b) Suppose $y = \frac{x^4}{x^2-3}$ . Find simpler functions $f(x), g(x)$ and $h(x)$ so that $y = \frac{f(x)}{g(x)h(x)}$.

   (c) Suppose $y = x(x^2-3) - 6x^3(x^2-3)$. Find simpler functions $f(x)$ and $g(x)$ so that $y = f(x)g(x)$.

   (d) Suppose $y = 4x(x+1) + 2(x^3+x^2)$. Find simpler functions $f(x)$ and $g(x)$ so that $y = f(x)g(x)$.

3. Let $u(t)$ be as in (2.6.4). Compute the multipart rules and graphs for $u(\frac{t}{2}-1)$ and $u(\frac{1}{2}(t-1))$. Explain how to obtain each function from $u(t)$ by applying the operations of §2.5; be very specific about the order in which you apply the operations.

4. Let $u(t)$ be the basic step function in (2.6.4). Find the multipart rules and sketch the graphs of:

   (a) $y = u(t) + u(t+2) - 3u(t-4)$.

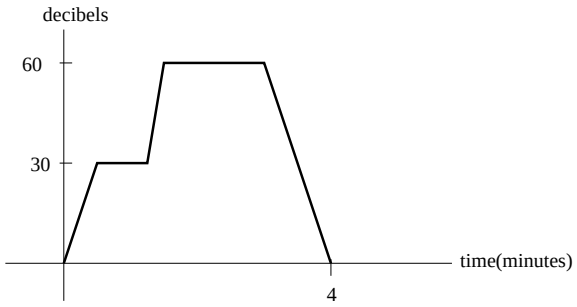
   (b) $y = u(t) + 2u(2t-4) - 2u(\frac{t}{2}+3)$.

5. Return to the cellular biology experiment explained at the beginning of this section. Describe how to model the voltage plots from experiments #1,2,3 using the basic step function $u(t)$ in (2.6.4).

6. Let $u(t)$ be the basic step function in (2.6.4). Suppose we start with a step function of the form $y = Au(\frac{1}{B}(t-C)) + D$, where $A, B, C, D$ are all non-negative constants with both $A$ and $B$ positive. (For example, $A = B = 1$, $C = D = 0$ is just $u(t)$; if $A = 1/2, B = 6, C = 4, D = 3$, we get the function in Example (2.6.6).) The graph of this function determines a rectangular region on the domain $C \le t \le C + B$. Sketch this region and compute it's area in terms of $A, B, C, D$.

   *Note:* This problem is related to a topic in cellular biology called *neuronal arithmetic*. The combination of electrical signals arriving at a nerve cell can be modeled by a sum of functions of this type. Adding up the corresponding areas is of physical significance since it is a measurement of "work" being performed.

7. Mike is a Pearl Jam fanatic and goes into a daze everytime he hears the first track of their latest CD. The loudness (in decibels) of this 4 minute track is modeled by the graph of the function $y = f_1(t)$ at time $t$ minutes; the graph is given below:




(a) Suppose Mike makes three seperate recordings of this track. Let $f_2(t), f_3(t), f_4(t)$ represent the loudness (in decibels) of this 4 minute track, with $f_2(t)$ 15% louder than $f_1(t)$, $f_3(t)$ 15% louder than $f_2(t)$ and $f_4(t)$ 15% louder than $f_3(t)$. Sketch the graph of each function on the domain $0 \leq t \leq 4$.

(b) Mike records this track over and over on a tape recorder so he can listen to it four times in a row. He has decided to make each successive recording 15% louder than the previous one. Sketch the graph of $m(t) = f_1(t) + f_2(t-4) + f_3(t-8) + f_4(t-12)$. What does this graph represent?