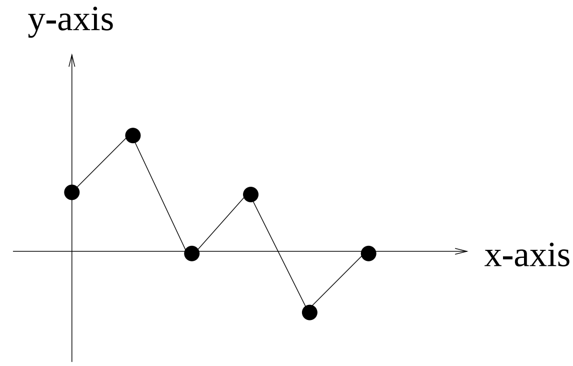
## 2.1 Graphical Analysis

1. The drawing shows the graph of a function $y = f(x)$, defined on the domain $0 \le x \le 5$. The graph connects the the six points pictured with line segments; the six points are $(0,1), (1,2), (2,0), (3,1), (4,-1), (5,0)$.



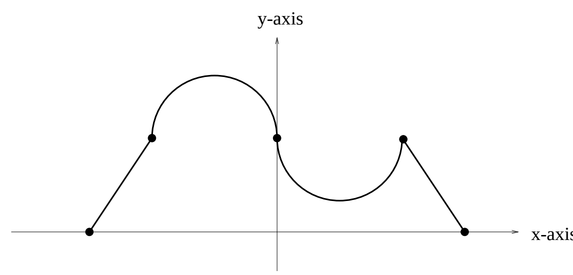

   (a) What is the range of $f$?

   (b) How many times does the graph of $f$ cross the line with equation $y = a$, with $a = 0, \frac{1}{2}, 1, 2, 3$?

   (c) Where is the function increasing? Where is the function decreasing? Where is the function positive? Where is the function negative?

   (d) Find the multipart formula for $y = f(x)$. Use this to find all solutions of the equation: $\frac{1}{2} = f(x)$.

2. The graph of a function $y = g(x)$ on the domain $-6 \le x \le 6$ consistes of line segments and semicircles of radius 2 connecting the points $(-6,0), (-4,4), (0,4), (4,4), (6,0)$.




   (a) What is the range of $g$?

   (b) Where is the function increasing? Where is the function decreasing?

   (c) Find the multipart formula for $y = g(x)$.

   (d) If we restrict the function to the smaller domain $-5 \le x \le 0$, what is the range?

   (e) If we restrict the function to the smaller domain $0 \le x \le 4$, what is the range?

3. Start with the multipart function

$$f(x) = \begin{cases} 1 & \text{if } x \le 0 \\ 1 - x & \text{if } 0 < x < 2 \\ -1 & \text{if } x \ge 2 \end{cases}$$

   (a) What is the domain of $f$?

   (b) Sketch the graph of $f$.

   (c) What is the range of $f$?

   (d) Where does the graph of $y = \frac{1}{2}x + \frac{1}{4}$ intersect the graph of $f(x)$? (Sketch the picture.)

   (e) Suppose a line $\ell$ of slope $m$ passes through the point $(-3,2)$. If $m = -1$, how many times does the line $\ell$ intersect the graph of $f$? If instead $m = \frac{-3}{5}$, how many times does the line $\ell$ intersect the graph of $f$? (Sketch the pictures.)

(f) Continue to work with the line $\ell$ in (e). For what values of $m$ will the line $\ell$ intersect the graph of $f$ at least two times?

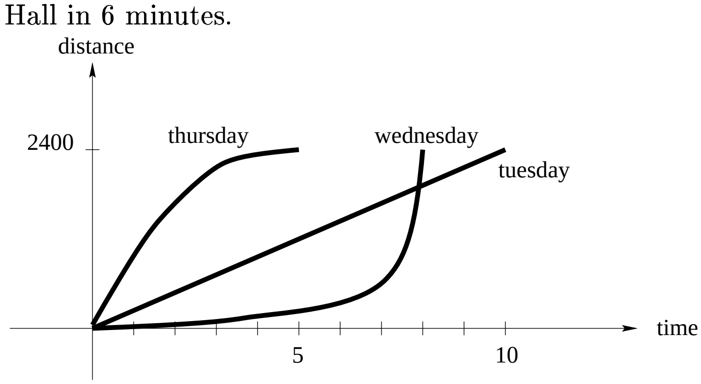
4. Dave leaves his office in Padelford Hall on his way to teach in Gould Hall. Below, are several different scenarios. In each case, sketch a plausible (reasonable) graph of the function $s = d(t)$ which keeps track of Dave's distance $s$ from Padelford Hall at time $t$. Take distance units to be "feet" and time units to be "minutes". Assume Dave's path to Gould Hall is along a straight line which is 2400 feet long.


(a) Dave leaves Padelford Hall walking slowly. But, his morning coffee buzz starts to kick in and his speed steadily increases (i.e. he walks faster and faster) until he reaches Gould Hall in 4 minutes.

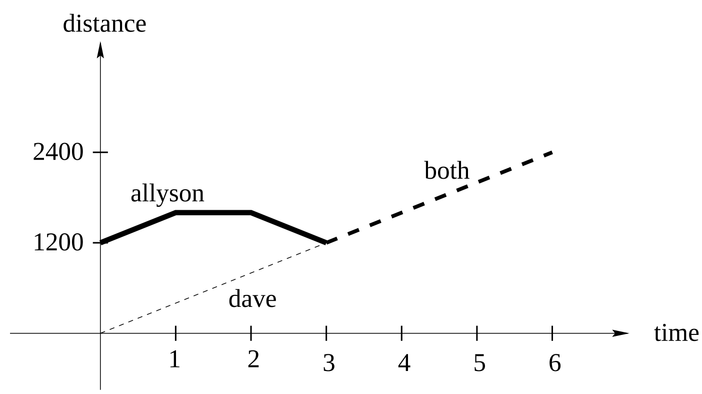
(b) Dave leaves Padelford Hall and his speed steadily increases (i.e. he walks faster and faster) until reaching the half-way point in 5 minutes. Dave is exhausted, so he stops to rest for 90 seconds. Then he continues on to Gould Hall, walking at a constant speed.

(c) Dave leaves Padelford Hall running, but his speed steadily decreases (i.e. he moves slower and slower) until reaching Gould Hall in 6 minutes.

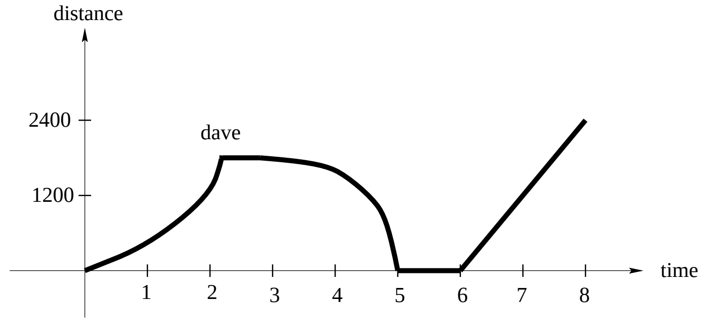
(d) Here is a plot of Dave's "distance from padelford" vs. "time" on Tuesday, Wednesday and Thursday of the first week of classes. Describe Dave's behavior in each case:



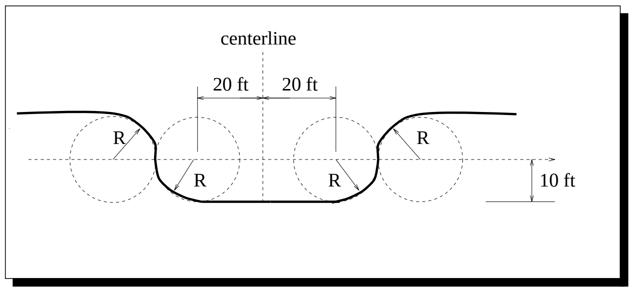

(e) The moment Dave leaves Padelford Hall, Allyson is located on his path. Allyson is also heading toward Gould Hall along Dave's path, but she is initially 1200 feet in front of Dave. Here is a plot of "distance from padelford" vs. "time" for both of them. Describe in words what is happening here.



(f) Here is a plot of Dave's "distance from padelford Hall" vs. "time". Describe in words what is going on.




5. The vertical cross-section of a drainage ditch is pictured below:




Here, $R$ indicates a circle of radius 10 feet and all of the indicated circle centers lie along the common horizontal line 10 feet above and parallel to the ditch bottom. Assume that water is flowing into the ditch so that the level above the bottom is rising 2 inches per minute.

(a) What is the width of the filled portion of the ditch after 1 hour and 18 minutes?

(b) When will the filled portion of the ditch be 42 feet wide? 50 feet wide? 73 feet wide?

(c) When will the ditch be completely full?

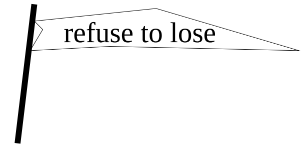
(d) Find a multipart function that models the vertical cross-section of the ditch.

6. (a) Let $\ell$ be the line passing through the two points $(12, -1)$ and $(-2, 1)$. Determine where $\ell$ crosses the circle of radius $r = 2$ centered at the point $(h, k) = (1, -1)$.

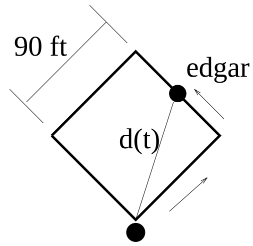
(b) A line of slope $m$ passes through the origin and a circle of radius 2 is centered at the point (10,0). Find all values of $m$ so that the line intersects the circle at least once.

7. An irrigation canal has a cross-section in the shape of a semicircle which is 10 feet deep at the center. Find the depth 4 feet from the edge of the canal.

8. This is your second day on the job as a sales representative for Big-Glo Inc., designers of a full line of new halogen light fixtures for use in rural areas. The boss has called you into the conference room to answer a question by one of the company investors. The question is this: Suppose a number of lights are installed in along a straight line and suppose each light casts a usable illuminated disc of radius 75 feet. If the distance between the successive poles is 90 feet, there will be an "overlap" effect. How can we figure out exactly where these illuminated discs are crossing one another?




9.*A baseball diamond is a square with sides of length 90 ft. Assume Edgar hits a home run and races around the bases (counterclockwise) at a speed of 18 ft/sec. Express the distance between Edgar and home plate as a function of time $t$. (Hint: This will be a multipart function.) Try to sketch a graph of this function.




10. (a) Graph the equations $y = x + 1$ and $y = -2x + 4$ on the domain of all real numbers. Use this picture to interpret the $x$-values which make the equation $x + 1 > -2x + 4$ true. Also solve the inequality symbolically.

    (b) Graph the equations $y = -x + 3$ and $y = 2 + \sqrt{4 - x^2}$ on the domain $-2 \leq x \leq 2$. Use this picture to interpret the $x$-values which make the equation $-x + 3 \leq 2 + \sqrt{4 - x^2}$ true. Also solve the inequality symbolically.

11. (a) Suppose $y$ is a linear function of the variable $t$; write down a formula for $y$.

    (b) Suppose $y$ is a linear function of the variable $\theta$; write down a formula for $y$. (Note: We pronounce the Greek letter $\theta$ as "*thā − tah*".)

12. Sketch a graph of the multipart function

$$f(x) = \begin{cases} 1 & \text{if } x \leq -1 \\ \sqrt{1 - (x+1)^2} & \text{if } -1 \leq x \leq 0 \\ -\sqrt{1 - (x-1)^2} & \text{if } 0 \leq x \leq 1 \\ -1 & \text{if } x \geq 1 \end{cases}$$

13. Return to Example 2.2.1 and answer these questions:

    (a) How much time does the pilot spend below the level of the glider port?

    (b) When is the pilot at maximum elevation?

    (c) When is the pilot at minimum elevation?

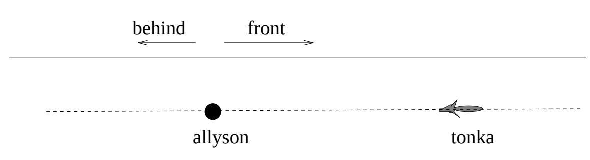
    (d) When does the pilot immediately go from climbing to diving?

    (e) When is the pilot 100 feet above the glider port?

    (f) How much time does the pilot spend under 200 feet above the surface of the ocean?

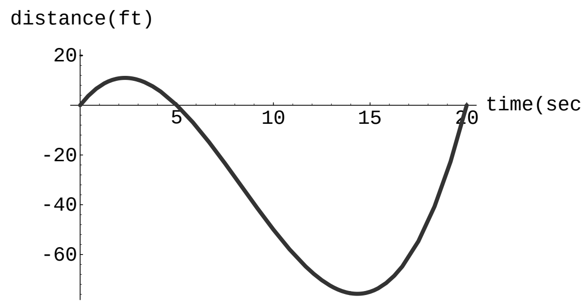
    (g) When is the pilot gaining or losing elevation at the largest rate?

14. Allyson is playing with her dog *Tonka* who runs forward and backward along a straight line, as pictured. Lee has gathered extensive data on Tonka's distance in front of Allyson at time $t$ seconds; note that a negative value for this distance means that Tonka is behind Allyson. He has concluded that the function $s = d(t) = \frac{1}{10}t^3 - \frac{5}{2}t^2 + 10t$ describes Tonka's location at time $t$ during the first 20 seconds.




TOP VIEW- not to scale

(a) Here is the graph of the function $s = d(t)$. Plot the points $P = (t, d(t))$, where $t = 0, 2, 4, 6, 8, \ldots, 20$ and indicate the coordinates of each point.




(b) Determine where the graph crosses the axes (both graphically and using algebra) and what is the physical meaning of these points in terms of the problem?

(c) At time $t = 3$ seconds, is Tonka within a distance of 10 feet of Allyson?

(d) At time $t = 10$ seconds, is Tonka a distance of more than 40 feet from Allyson?

15. This exercise deals with visualizing the domain and range of a function.

    (a) Let $y = f(x) = x + 1$. Graphically relate the domain, graph and range for the following domains: $0 \leq x \leq 10$, $-10 \leq x \leq 0$, $-2 \leq x \leq 2$.

    (b) Let $y = f(x) = -\frac{1}{2}x - 4$. Graphically relate the domain, graph and range for the following domains: $0 \leq x \leq 10$, $-10 \leq x \leq 0$, $-2 \leq x \leq 2$.

    (c) Find a linear function $y = mx + b$, some numbers $m, b$, so that the domain $-1 \leq x \leq 1$ yields the range of $2 \leq y \leq 5$. Can you find two different linear functions with this property?

    (d) Let $y = f(x) = 2 - \sqrt{4 - (x+1)^2}$. Find the largest possible domain for which this function is defined. Graphically relate the domain, graph and range.

16. For each case a.-g., sketch the graph of a function $y = f(x)$ satisfying the specified conditions. Make your graphs *connected*; that means you can draw the curve without picking up your pencil (there are no jumps or holes in the graph). Note, there are many different correct answers.

    (a) Domain all real numbers, function positive if $x$ is negative and negative if $x$ is positive.

    (b) Domain all real numbers, function negative if $x$ is negative and positive if $x$ is positive.

    (c) $f(0) = 1, f(-1) = 2, f(1) = 0$, domain all real numbers.

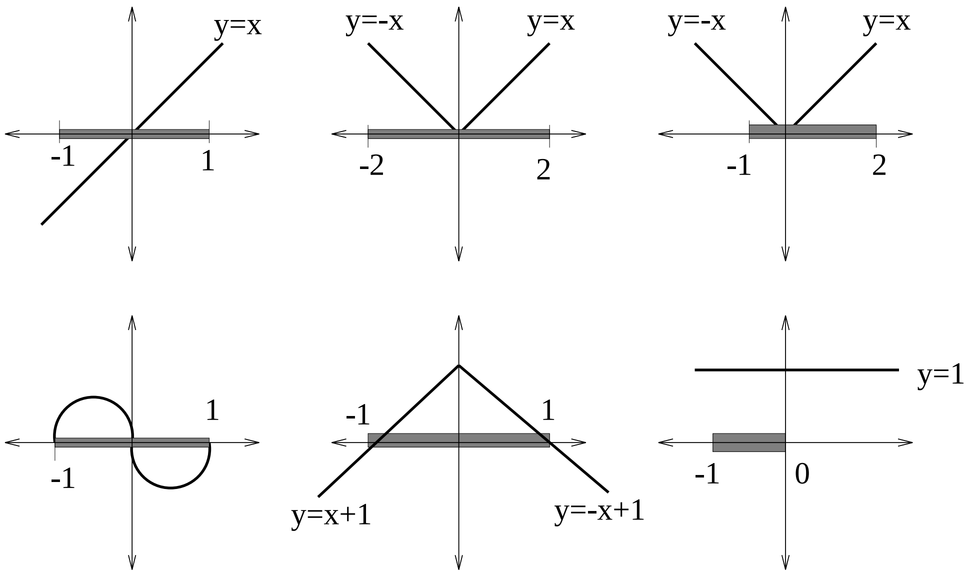
    (d) Domain all real numbers, the function always has positive values, the function is decreasing if $x$ is negative and increasing if $x$ is positive.

    (e) Domain all real numbers, the graph crosses the line $y = 1$ when $x = 0, 1, 2, 3, 4$.

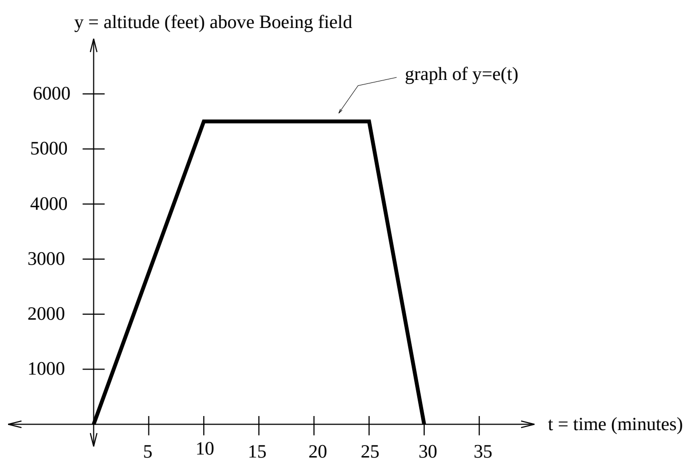
    (f) Domain=$0 \leq x \leq 1$, range=$-1 \leq y \leq 2$.

    (g) Domain all real numbers, range=all positive numbers.

17. Below we have pictured a number of graphs and indicated a domain on the horizontal axis (the shaded box). For each graph, discuss: (1) the range; (2) where the function is positive, negative and zero; (3) where the function crosses the axes; (4) where the function is increasing and decreasing.




18. An airplane takes off from Boeing field. The aircraft climbs at a steady rate of 550 feet/minute for 10 minutes, then flies at a level altitude for 15 minutes. Finally, during the time $t$ between 25 and 30 minutes after takeoff, the plane descends at a constant rate and lands at Boeing field at time $t = 30$ minutes. Let $e(t)$ be the function which describes the altitude of the airplane above Boeing field $t$ minutes after takeoff. Here is a sketch of the graph of $e(t)$.




(a) Find an explicit formula for the function $y = e(t)$ during the first 10 minutes of flight.

(b) Find an explicit formula for the function $y = e(t)$ during the time between 10 and 25 minutes after takeoff.

(c) During the last 5 minutes of the flight, find an explicit formula for the function $y = e(t)$. At what rate does the plane descend during the last 5 minutes of the flight?

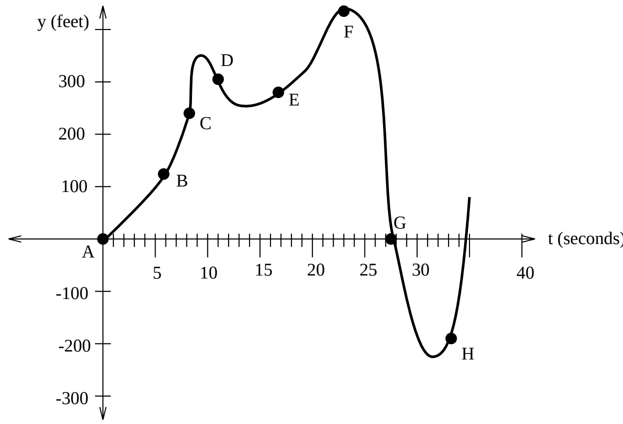
(d) During the time between 20 and 27 minutes after takeoff, what are the possible altitudes of the airplane?

(e) From part (a) you should get that "$e(7) = 3850$"; describe what this equation means in words.

(f) Assume a hot air balloon is drifting 1000 feet above the elevation of Boeing field. How much time does the plane spend above the balloon?

(g) Let $b(t)$ be the altitude of the plane above the hot air balloon at time $t$; sketch a graph of $b(t)$.

19. Angela likes to spend her weekend hang-gliding. Here is a plot of the function $y = h(t)$ which gives her height (in feet) above the gliderport at time $t$ (seconds) after launch.




(a) Between which pairs of consecutive points is the graph always increasing or always decreasing?

(b) About how much time does Angela spend ascending?

(c) Between which pairs of consecutive points does Angela have the largest change in height?

(d) The slope of a line segment connecting successive points on the graph is called the *average velocity* during the corresponding time period. Using the points $A, B, \ldots, H$, answer these questions: When does Angela have the largest positive and negative average velocity? When does Angela have average velocity closest to zero?

20. *Hemoglobin* is a very important protein molecule found in human red blood cells. One duty of hemoglobin is to bind itself to oxygen $O_2$. Each hemoglobin molecule can bind to a maximum of four $O_2$ molecules; when this happens we say the hemoglobin is *saturated*. The extent to which hemoglobin is saturated is a function of the oxygen pressure $p$, measured in units called *torrs*; 760 torr= 1 atmosphere. The fraction $y$ of saturated hemoglobin molecules in an average red blood cell is given by the function

$$y = s(p) = \frac{p(1+p)^3 + 126p(1+0.014p)^3}{(1+p)^4 + 9000(1+0.014p)^4}$$

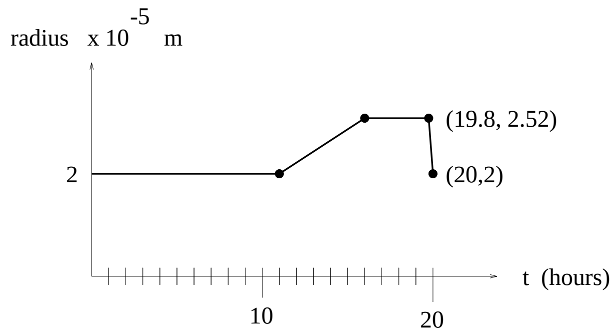
(a) Use a graphing device to plot the graph of $s(p)$; you will need to settle on a reasonable viewing window.

(b) Describe in words what the graph tells you about the relationship between hemoglobin saturation and pressure.

(c) Use your graphing device to determine the pressure required so that 90% of the hemoglobin is saturated.

(d) A patient has a disease in which the bodies ability to maintain $O_2$ pressure in the red blood cells declines with age. At age 30 the patient can maintain a pressure $p = 100$ torr, but this decreases with age at the rate of 5 torr every 2 years. The patient has asked you when he will only be able to saturate half of his hemoglobin; what is your reply?

21. There is a very delicate relationship between the volume and surface area of a cell. Here is the reason why: The rate waste is produced by a cell is proportional to its volume. But, the rate waste can be expelled from a cell is proportional to its surface area. (This is because waste products must pass from the inside of the cell across the *cell membrane* to get outside the cell. Thinking of a cell as a sphere, the surface of the sphere is like the cell membrane.) Suppose a particular mouse cell is spherical. During a 20 hour observation period, the average radius of a population of these cells is given by the function $r(t)$ graphed below:

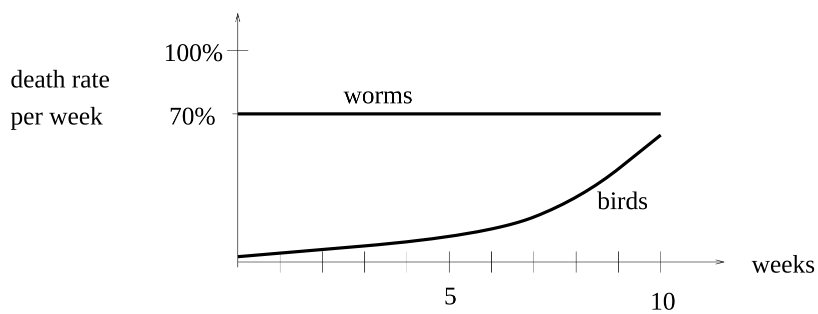



(a) Write out a multipart rule for $r(t)$.

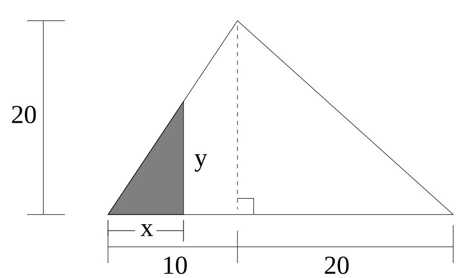
(b) Let $S(t)$ and $V(t)$ denote the surface area and volume of the cell at time $t$ hours; $t = 0$ corresponds to the instant you start making your observations. Write out the multipart rule for the *new* function $d(t) = \frac{S(t)}{V(t)}$.

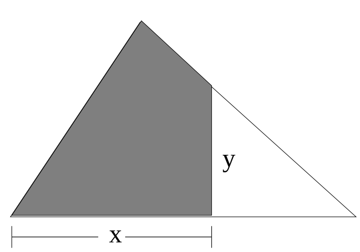
(c) Using a graphing device, sketch a graph of $d(t)$ for $0 \le t \le 20$ hours. Using this graph, describe how the cells ability to expel waste is changing.

22. An ecologist is performing an experiment. Into a controlled environment, 1000 worms and 1000 birds are introduced. The graph below indicates the death rate of the two species, as a function of time in units of weeks. Use this information to sketch a *new* graph of the *survivorship curve* for each species; i.e. plot the number of survivors (for each species) versus time.




23. Pizzeria Buonapetito makes a triangular-shaped pizza with base width of 30 inches and height 20 inches as shown. Alice wants only a portion of the pizza and does so by making a vertical cut through the pizza and taking the shaded portion. Letting $x$ be the bottom length of Alice's portion and $y$ be the length of the cut as shown, answer the following questions:




shaded region is the piece Alice takes

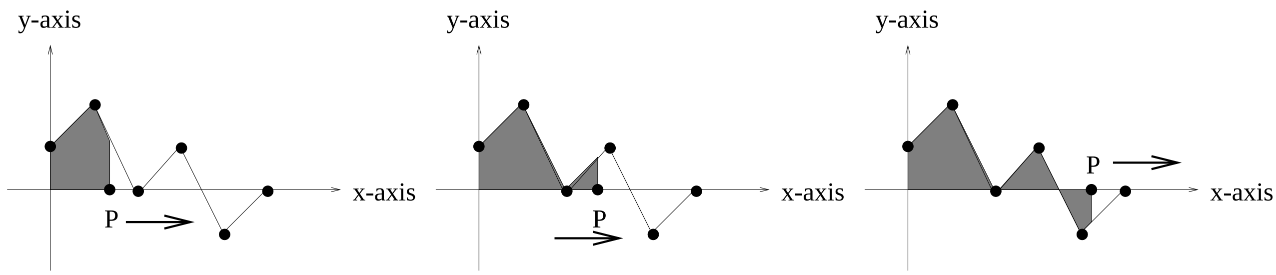
(a) Find a formula for $y$ as a multipart function of $x$, for $0 \le x \le 30$. Sketch the graph of this function and calculate the range.

(b) Find a formula for the area of Alice's portion as a multipart function of $x$, for $0 \le x \le 30$. Sketch the graph of this function and calculate the range.

(c) If Alice wants her portion to have half the area of the pizza, where should she make the cut?

(d) If Alice wants her portion to have $p\%$ of the area of the pizza, where should she make the cut?

24. This problem builds on Exercise 1 in this section. Recall, we have a function $y = f(x)$, defined on the domain $0 \le x \le 5$. The graph connects the the six points pictured with line segments; the six points are $(0,1), (1,2), (2,0), (3,1), (4,-1), (5,0)$. In this problem, we assume a point $P$ on the $x$-axis is moving to the right and is located at $(\frac{1}{2}t, 0)$ after $t$ seconds. As the point $P$ moves to the right, it can be used to construct a shaded region between the $x$-axis and the graph. We have pictured the three possible cases that will arise, depending on the exact location of $P$. Find a multipart function $A(t)$ that computes the area of the shaded region as a function of time $t$.




25. A course uses four scores to determine your final grade. These scores are numbers between 0 and 100; one each for homework $H$, quizzes $Q$, midterm exams $M$ and a final exam $F$. The professor first calculates your *course percentage* $p$, based on this information:

- Homework $H$ is worth 10%.
- Quizzes $Q$ are worth 20%.
- Midterms $M$ are worth 30%.
- Final $F$ is worth 40%.

(a) Find an equation involving $H, Q, M, F$ for calculating your course percentage $p$. If your scores are $H = 68, Q = 90, M = 84, F = 88$, what is your course percentage?

(b) Assume these facts:

- A course grade is either 0 or a decimal number between 0.7 and 4.0.
- A course percentage of $p = 96$ gives a grade of 4.0 and a course percentage of $p = 43$ gives a grade of 0.7. Between these two percentages, the grade is given by a linear function.

Find a multipart function that calculates the grade based on the course percentage $p$. Make sure to specify the domain and range of this function.

(c) If your scores are $H = 68, Q = 90, M = 84, F = 88$, what is your final grade?

(d) What course percentage is needed to get a grade of 3.7?

(e) Assume the number $N$ of students having a course percentage of $p$ is given by the function

$$N = -0.015p(p - 100).$$

How many students will get a grade of at least 3.7?

26. Your biology professor announces a two step procedure for calculating your midterm grade. First, the numerical grade $G$ for your midterm and your midterm exam score of $S$ (a score between 0 and 100) must give a solution to the equation: $100G = 5S - 50$. Secondly, University policy requires that the numerical grade $G$ is either 0 or a number in the interval $0.7 \leq G \leq 4.0$. In other words, grades of 4.1, 0.5, 0.3, etc. are not assigned.

   (a) Find a multipart function $G = b(S)$ which computes the biology grade for a given score.

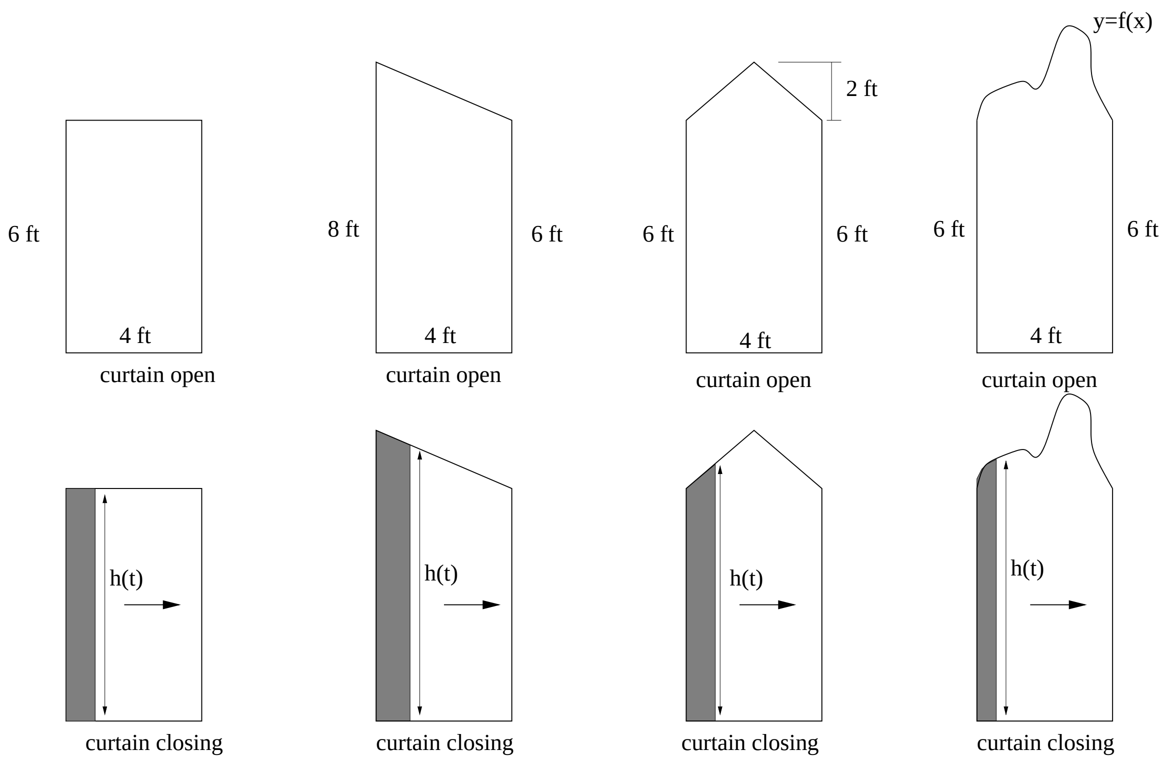
   (b) Draw a picture of the graph of this function.

   (c) If you score 82 on the exam, what is your biology grade?

   (d) What exam scores lead to a biology grade of 4.0?

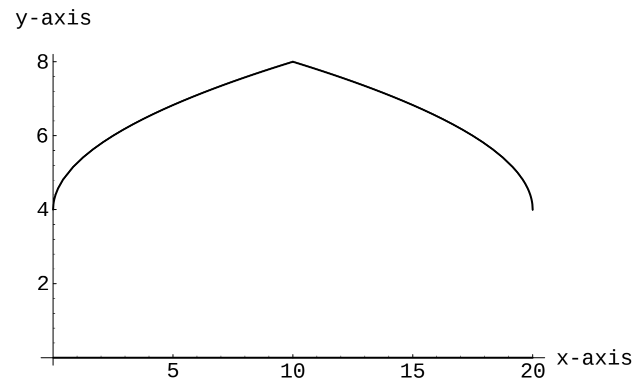
   (e) What exam scores lead to a biology grade of 0?

27. A vertical curtain moves across a window opening from left to right, moving 6 inches/sec. For each of the window shapes pictured below, find a function $h(t)$ that calculates the height of the right vertical edge of the curtain at time $t$ seconds after it begins to close. In each case, specify the domain and range of the function.



28. The top profile of an archway is given by the graph of the multipart function $y = f(x)$ graphed below; axes units are in feet and the centerline is $x = 10$. What is the maximum height of a person who can walk under the archway 5 feet right of center. What is the maximum height of a person who can walk under the archway 3 feet left of center?

$$f(x) = \begin{cases} 4 + 4\sqrt{x/10} & \text{if } 0 \le x \le 10 \\ 4 + 4\sqrt{(20-x)/10} & \text{if } 10 \le x \le 20 \end{cases}$$




29. This problem deals with cars traveling between Bellevue and Spokane, which are 280 miles apart. Let $t$ be the time in hours, measured from 12:00 noon; so, for example, $t = -1$ is 11:00 am.

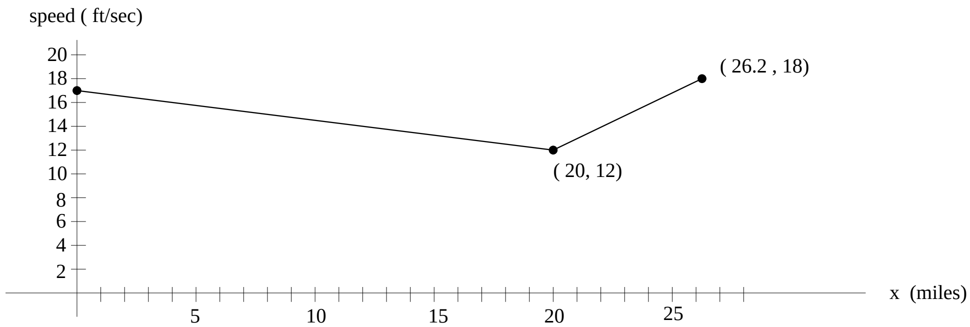
   (a) Joan drives from Bellevue to Spokane at a constant speed, departing from Bellevue at 11:00 am and arriving in Spokane at 3:30 pm. Find a function $j(t)$ that computes her distance from Bellevue at time $t$. Sketch the graph, specify the domain and determine the range.

   (b) Steve drives from Spokane to Bellevue at 70 mph, departing from Spokane at 12:00 noon. Find a function $s(t)$ for his distance from Bellevue at time $t$. Sketch the graph, specify the domain and determine the range.

   (c) Find a function $d(t)$ that computes the distance between Joan and Steve at time $t$. (Caution: This will be a multipart function.) Sketch the graph, specify the domain and determine the range.

30. You have just taken a midterm exam in your *Precalculus* class and your *Chemistry* class. Each professor has announced a grading scale to compute your grade $G$ (a number between 0 and 4), given your midterm exam score $S$, where $0 \le S \le 100$. In the *Precalculus* class, the grade is computed by considering three different cases: (i) if your score S is equal to or over 88, then your grade is 4.0; (ii) if your score S is below 51, then your grade is 0; (iii) if your score S is between 51 and 88, then your grade is computed by a linear equation which assigns the grade of 0.7 to a score of 51 and the grade of 4.0 to a score of 88. In the *Chemistry* class, the grade is computed by considering three different cases: (i) if your score S is equal to or over 94, then your grade is 4.0; (ii) if your score S is below 34, then your grade is 0; (iii) if your score S is between 34 and 94, then your grade is computed by a linear equation which assigns the grade of 0.7 to a score of 34 and the grade of 4.0 to a score of 94. Find an explicit formula for each grading scheme. Determine the grade for a midterm score of 83 in each class. Determine when, if ever, a midterm score would yield the same grade in both classes. Sketch the graphs of all equations of interest in this problem.

31. Marathon runners like to keep a careful listing of their performance during the 26.2 mile race. Here is a plot of Tim's speed (in units of "feet/sec") at mile $x$ during a marathon. He starts the race running 17 ft/sec. This graph consists of two line segments:




Let $s(x)$ be the function which tells us the runners speed at mile $x$ in the race.

(a) What is a formula for $s(x)$ during the first 20 miles of the race?

(b) What is a formula for $s(x)$ during the last 6.2 miles of the race?

(c) What is Tim's speed at mile 12?

(d) During what portion(s) of the race is Tim's speed greater than 15 feet/sec?

(e) Runners tend to think in terms of "pace", which is defined to be the number of minutes required to run one mile. (Olympic athletes can maintain a 5 min/mile pace.) What is Tim's pace in "min/mile" at mile 12?