1.4 Linear Modeling

1. This exercise emphasizes the "mechanical aspects" of working with linear equations:
   (a) Find the equation of a line passing thru the points $(1, -1)$ and $(-2, 4)$.
   (b) Find the equation of a line passing thru the point $(-1, -2)$ with slope $m = 40$.
   (c) Find the equation of a line with y-intercept $b = -2$ and slope $m = -2$.
   (d) Find the equation of a line passing thru the point $(4, 11)$ and having slope $m = 0$.
   (e) Find the equation of a line perpendicular to the line in a. and passing thru (1,1).
   (f) Find the equation of a line parallel to the line in b. and having y-intercept $b = -14$.
   (g) Find the slope of the line having equation $3x + 4y = 7$.
   (h) Find the equation of a line crossing the x-axis at $a = 1$ and having slope $m = 1$.

2. This problem emphasizes the "mechanical aspects" of working with the quadratic formula:
   (a) Solve for t: $2 = \sqrt{(1+t)^2 + (1-2t)^2}$.
   (b) Solve for t: $\frac{3}{\sqrt{5}} = \sqrt{(1+t)^2 + (1-2t)^2}$.
   (c) Solve for t: $0 = \sqrt{(1+t)^2 + (1-2t)^2}$.
   (d) Solve for x: $5x^2 = 6x$.
   (e) Solve for x: $5x^2 = 6x + 2$.
   (f) Solve for x: $5x^2 = 6x - 2$.
   (g) Assume h is constant and solve for a: $ha^2 + 4ha - 7h^2 = 1$.
   (h) Assume a is constant and solve for h: $ha^2 + 4ha - 7h^2 = 1$.
   (i) Assume a, b are constants, solve for x: $ax^2 + bx = 0$.
   (j) Assume a, h are constants, solve for x: $ax^2 - 2ahx + ah^2 = 0$.

3. While speaking to a friend in Oslo, Norway, you learned that the current temperature was -23^o Celsius (^oC). After the phone conversation, you wanted to convert this temperature to Fahrenheit degrees ^oF, but you can not find a reference with the correct formulas. You then remember that the relationship between ^oF and ^oC is linear. Using this and knowledge that 32 ^oF=0 ^oC and 212 ^oF = 100 ^oC, find an equation which computes Celsius temperature in terms of Fahrenheit temperature; i.e. an equation of the form C= "an expression involving only the variable F". Likewise, find and equation which computes Fahrenheit temperature in terms of Celsius temperature; i.e. an equation of the form F= "an expression involving only the variable C". How cold was it in Oslo in ^oF?

4. *Fairville, Oregon* mailed out its 1994 real estate tax bills to all property owners. Your neighbor to the south (Mr. Hillouly) lives on a 5500 square foot lot and received a tax bill for \$2856; your neighbor to the north (Ms. Hardin) living on a 3500 square foot lot received a tax bill for \$1856. Your tax bill for a 8700 square foot property is \$6723.

   The information accompanying the bill states "*... the tax on a property follows a linear model relating the area x (in square feet) and the tax bill \$$y$ ...*" The actual equation relating x and y is omitted and when you call the county assessors office you discover that they have lost it!

   You are convinced your bill is in error and the county assessor has scheduled a meeting for you to argue the case.

(a) Prepare a mathematically convincing argument that either your bill or one of your two neighbors bills is in error.

(b) Assuming that the bills for Mr. Hillouly and Ms. Hardin are correct, find your corrected tax bill.

5. (a) Using the first and last data points in Table 1.4.1, show the women's earning power linear model has equation

$$y = \frac{12915}{17}(x - 1970) + 5616.$$

Sketch the graph modeling the years between 1970 and 2000. What is women's earning power in 1995? When will women's earning power be \$21,000?

(b) Recall the linear models for Men and Women's earning power in (1.4.1). The "women's percentage of men's earnings" is given by the formula

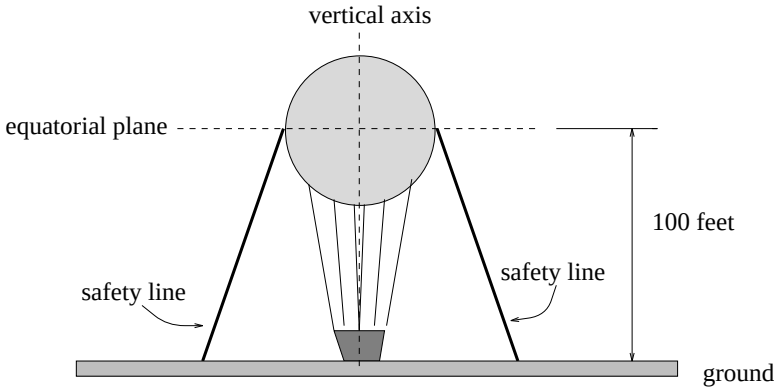
$$p = \frac{\text{women}'\text{ssalary}}{\text{men}'\text{ssalary}} \times 100.$$

What is this percentage in 1996? When is $p = 68\%$? Is the percentage ever $p = 86\%$? What is happening to p as we look farther and farther into the future?

(c) Plot the graphs of the men's and women's earning power equations in the same coordinate system. Use this graph to explain whether or not women's earning power is improving with respect to men's earning power. What aspect of each equation controls whether women are gaining on men?

6. Find the coordinates of the two points P and Q where the line having equation $y = -\frac{1}{2}x + 1$ intersects the circle of radius 1 centered at (-2,2). Then find the perimeter of the triangle with vertices: P, Q and $(-4, 0)$. Draw an accurate picture of the triangle being studied.

7. A spherical balloon of radius 30 feet is to be anchored to the ground by two lines as pictured below:



USBA Safety regulation #TD-83-7856a

The U.S. Balloon Association safety regulations require that the safety lines have a slope of absolute value $\frac{3}{2}$ and be anchored symmetrically with respect to the basket location on the ground. Find the length of each safety line and where it must be anchored to the ground.

8. In 1990 you purchase a house for \$120,000. After 5 years the house is valued at \$234,000. Assume that the future value of the house and the number of years the house is owned are linearly related.

(a) Find an equation which relates the value v of the house and t years of ownership.

(b) What is the value of the house after you have owned it for 24 years?

(c) How long must you own the house for it to be worth \$360,400?

9. The (average) sale price for single family property in Seattle and Port Townsend is tabulated below:

YEAR	SEATTLE	PORT TOWNSEND
1970	\$38,000	\$8400
1990	\$175,000	\$168,400

(a) Find a linear model relating the year x and the sales price y for a single family property in Seattle.

(b) Find a linear model relating the year x and the sales price y for a single family property in Port Townsend.

(c) Sketch the graph of both modeling equations in a common coordinate system; restrict your attention to $x \geq 1970$.

(d) What is the sales price in Seattle and Port Townsend in 1983 and 1998?

(e) When will the average sales price in Seattle and Port Townsend agree and what is this price?

(f) When will the average sales price in Port Townsend be \$15,000 less than the Seattle sales price? What are the two sales prices at this time?

(g) When will the Port Townsend sales price be \$15,000 more than the Seattle sales price? What are the two sales prices at this time?

(h) When will the Seattle sales price be double the Port Townsend sales price?

(i) Is the Port Townsend sales price ever double the Seattle sales price?

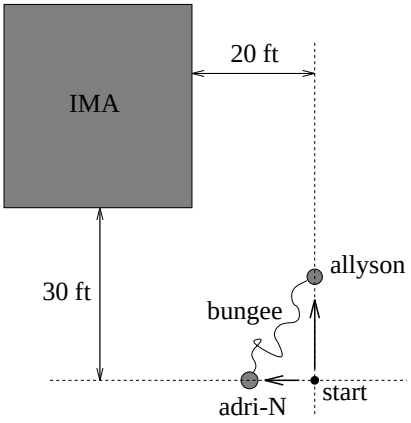
10. Consider the equation: $\alpha x^2 + 2\alpha^2 x + 1 = 0$. Find the values of x that make this equation true (your answer will involve α). Find values of α that make this equation true (your answer will involve x).

11. The yearly tuition at *Hometown University* was \$1000 in 1989 and \$2700 in 1995. Find a linear model which fits this data. When will the tuition at Hometown University coincide with the tuition at the University of Washington, using the model in Example 1.4.13? When will the tuition at Hometown University be \$5000 per year more than the UW tuition?

12. Return to Example 1.4.17. When is the duster closest to the center of the field?

13. In problem 1(a)-(f) you found equations of the form "y=an expression involving the variable x". Rewrite each equation in the form "x=an expression involving the variable y".

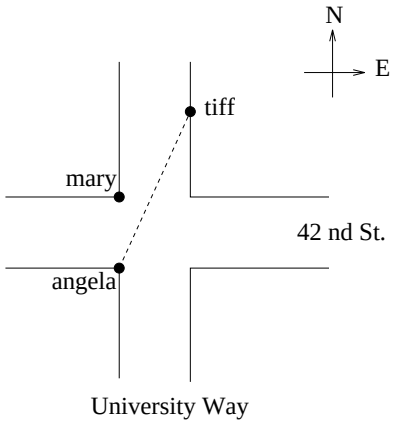
14. Allyson and Adrienne have decided to connect their ankles with a *bungee cord*; one end is tied to each persons ankle. The cord is 30 feet long, but can stretch up to 90 feet. They both start from the same location. Allyson moves 10 ft/sec and Adrienne moves 8 ft/sec in the directions indicated. Adrienne stops moving at time $t = 5.5$ sec, but Allyson keeps on moving 10 ft/sec in the indicated direction.



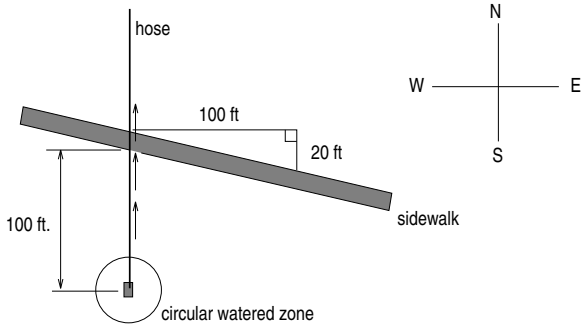
(a) Sketch an accurate picture of the situation at time $t = 7$ seconds. Make sure to label the locations of Allyson and Adriene; also, compute the length of the bungee cord at $t = 7$ seconds.

(b) Where is Allyson when the bungee reaches its maximum length?

15. Angela, Mary and Tiff are all standing near the intersection of University and 42^{nd} streets. Mary and Tiff do not move, but Angela runs toward Tiff at 12 ft/sec along a straight line, as pictured. Assume the roads are 50 feet wide and Tiff is 60 feet north of the nearest corner. Where is Angela located when she is closest to Mary and when does she reach this spot?



16.* The infamous *crawling tractor sprinkler* is located as pictured below, 100 feet South of a 10 ft. wide sidewalk; notice the hose and sidewalk are not perpendicular. Once the water is turned on, the sprinkler waters a circular disc of radius 20 feet and moves North along the hose at the rate of $\frac{1}{2}$ inch/second.

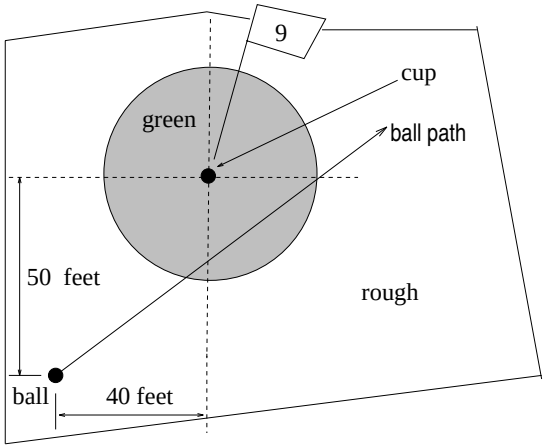


(a) Impose a coordinate system. Describe the initial coordinates of the sprinkler and find the equation of the line forming the southern boundary of the sidewalk.

(b) After 33 minutes, sketch a picture of the wet portion of the sidewalk; find the length of the wet portion of the Southern edge of the sidewalk.

(c) Find the equation of the line forming the northern boundary of the sidewalk. (Hint: You can use the properties of right triangles.)

17. The cup on the 9^{th} hole of a golf course is located dead center in the middle of a circular green which is 70 feet in diameter. Your ball is located as in the picture below:



The ball follows a straight line path and exits the green at the right-most edge. Assume the ball travels 10 ft/sec.

(a) Where does the ball enter the green?

(b) When does the ball enter the green?

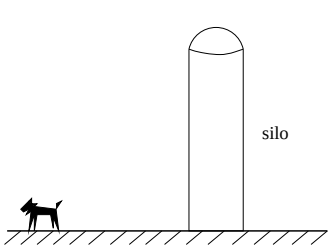
(c) How long does the ball spend inside the green?

(d) Where is the ball located when it is closest to the cup and when does this occur.

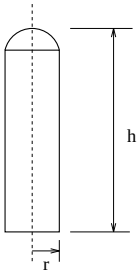
18. Complete the following table; in many cases there may be several possible correct answers:

Equation	slope	y-intercept	point on line	point on line
$y = 2x + 1$				
			(3,-4)	(-1,7)
	-2	1		
	$\frac{1}{2}$		(0,1)	
		1000		
	0			
			(3,3)	(3,-2)
			(5,-9)	

19. *Dave is going to leave academia and go into business building *grain silos*. A grain silo is a cylinder with a hemispherical top; a silo is used to store grain for farm animals. Here is a 3D view and a cross-section:

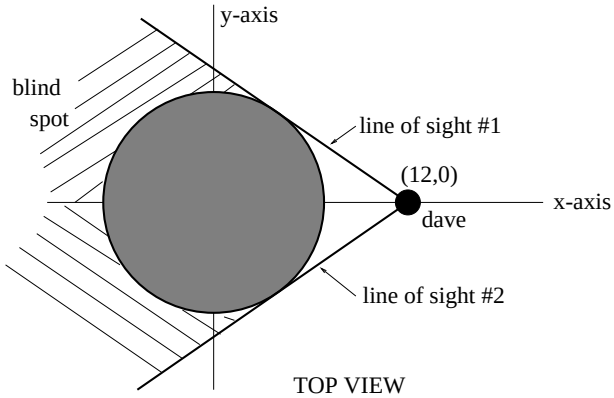


3D- view

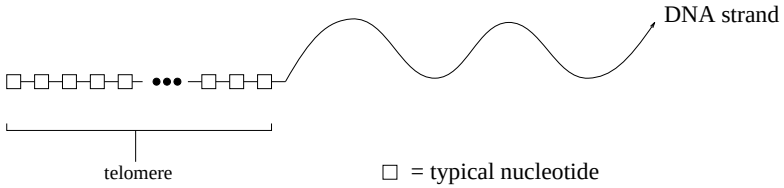


cross-section

If Dave is standing next to a silo of cross-sectional radius 8 feet at the indicated position, his vision will be partially obstructed. Find the portion of the y-axis that Dave can see. (Hint: Let a be the x-coordinate of the point where line of sight #1 is tangent to the silo; compute the slope of the line using two points (the tangent point and (12,0)). On the other hand, compute the slope of line of sight #1 by noting it is perpendicular to a radial line through the tangency point. Set these two calculations of the slope equal and solve for a.)



20. A mouse DNA molecule is made by bonding together a (long) sequence of building blocks called *nucleotides*. There are four possible nucleotides, which we will denote by the letters **A,G,C,T**. The ends of DNA contain repeating sequences of nucleotides called *telomeres*.



For example, we might have a telomere that looks like this:

GGGATTGGGATTGGGATTGGGATT...GGGATT,

so the sequence **GGGATT** is repeating itself. For a particular mutant mouse, as it ages, pieces of the telomeres are lost; i.e. an old mutant mouse has shorter DNA than a young mutant mouse.

When a mutant mouse was born, a geneticist measured that a telomere on a particular piece of DNA repeated the sequence **GGGATT** 3000 times. At age 2, a geneticist found this same telomere was 795 nucleotides shorter.

(a) What is the rate at which the telomere is shortening? Make sure to clearly indicate the units you are using.

(b) Find a linear model for the length of the telomere in terms of the mouse age.

(c) At what age will the telomere be gone?

Biologists are actively researching the connection between telomere length and the aging process.

21. After surviving Math 120 in Autumn 1996, Emile has decided to write and sell a booklet called *The Math 120 Student Survival Guide*. A friend has done some marketing analysis which shows that if he charges \$12, he will sell 500 copies. But, for each \$0.25 he lowers the price, sales will increase by an additional 40 copies. (For example, if he charges \$11.75, he will sell 540 copies; if he charges \$11.50, he will sell 580 copies, etc.) How many copies will Emile sell if the price is \$9.15? If Emile sold 1040 copies, what was the price?

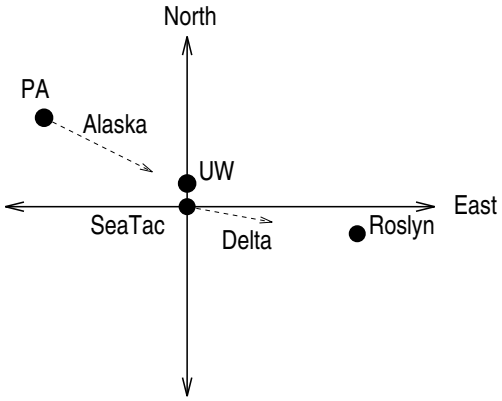
22. A car traveling on a freeway consumes gas at a constant rate. When the car has gone 100 miles there are 10 gallons of gas in the tank, and when it has gone 220 miles there are 6 gallons left.

    (a) Find a formula that expresses the amount F of gas in the tank when the car has gone x miles.

    (b) How much gas was in the tank when the car started on the trip?

    (c) How far has the car gone when it runs out of gas?

23. A Delta airlines jet flying 300 mph passes over SeaTac airport at 1:00pm, heading along a straight line course toward Roslyn, WA. Roslyn is located 65 miles East and 17 miles South of SeaTac. Also at 1:00pm, an Alaska airlines jet flying 400 mph passes over Port Angeles, heading toward the UW along a straight line course pictured. PA is located 62 miles West and 52 miles North of SeaTac and the UW is 15 miles due North of SeaTac.



    (a) Impose a coordinate system with SeaTac as the origin. Label the coordinates of Roslyn, PA, SeaTac and the UW in this coordinate system. (Use units of miles for each axes.)

    (b) Find the equations of lines modeling the path of the Delta jet and the Alaska jet.

    (c) Where do the two flight paths cross? Find the coordinates and label in the picture.

    (d) Is the Alaska jet located East or West of UW the instant the Delta jet passes over Roslyn?

    (e) Where is the Alaska jet located when it is closest to Roslyn (find the coordinates)? What is this minimum distance?

    (f) Where is the Alaska jet located (coordinates) when it is 88 miles from PA?

24. Explain why parallel lines have equal slopes; this is the first statement in (1.4.14).

25. Let ℓ_1 and ℓ_2 be two non-vertical lines passing through the origin. Assume the slope of ℓ_1 is m_1 and the slope of ℓ_2 is m_2. This exercise will outline how to show the following fact: Line ℓ_1 is perpendicular to line ℓ_2 exactly when $m_1 m_2 = -1$. Notice, the equation "$m_1 m_2 = -1$" is the same as the equation "$m_1 = -1/m_2$", which means the slopes are negative reciprocals of one another.

(a) Explain why ℓ_1 passes through the point $(1, m_1)$ and ℓ_2 passes through the point $(1, m_2)$.

(b) Sketch a picture of the triangle with vertices $(0,0), (1, m_1), (1, m_2)$.

(c) Assume our two lines are perpendicular. Using b., at which vertex is the right angle? Use the Pythagorean Theorem to obtain an equation which ultimately simplifies to $m_1 m_2 = -1$; which means the slopes are negative reciprocals of one another.

(d) Assume that $m_1 m_2 = -1$ and show that the two lines are perpendicular. (Hint: Assume the fact that a triangle whose sides a, b, c satisfy the equation $a^2 + b^2 = c^2$ is a right triangle with hypotenuse c.)

(e) Explain why all this work establishes the second statement in (1.4.14). The key point is that a.-d. only show this for lines passing through the origin.

26. Draw the graphs of $x = -1$ and $y = 2$ in the xy coordinate system. An ant starts at the location (6,2) and moves to the left along the line $y = 2$. Assume the position of the ant after t seconds is the point $P(t) = (6 - 2t, 2)$. At the same instant, a spider starts at the location (-1,-3) and moves upward along the line $x = -1$. Assume the position of the spider after t seconds is the point $Q(t) = (-1, -3 + t)$. In Exercise 1.3.18, we found the distance between the ant and spider at time t was given by the formula: $d(t) = \sqrt{5t^2 - 38t + 74}$. When is the distance between the two bugs exactly 4 units? Where are the bugs located at these times?

27. Sketch the circle of radius 4 centered at the point (2,3) and the graph of the line $y = -x + 2$.

(a) Determine the points where the line and circle intersect.

(b) An object starts at the location (-3,5) and moves along the line. Assume the position of the object at time t seconds is given by the formula $P(t) = (-3 + t, 5 - t)$. Plot the position of the object when $t = 0, 2, 4, 8$ seconds.

(c) Give a reason why the point $P(t)$ is always on the graph of the line $y = -x + 2$.

(d) When does the object enter and exit the circular zone?

(e) Find a formula $d(t)$ for the distance between the object and the origin at time t seconds. Your answer will involve t. Calculate $d(0), d(2), d(8)$ and explain what these numbers mean in the picture.

(f) When is the object 3 units from the origin? Where is the object at this time?

28. In Example 1.4.10, the linear equation $s = 24 - t$ was obtained. Explain the meaning of the minus sign within the context of that problem.