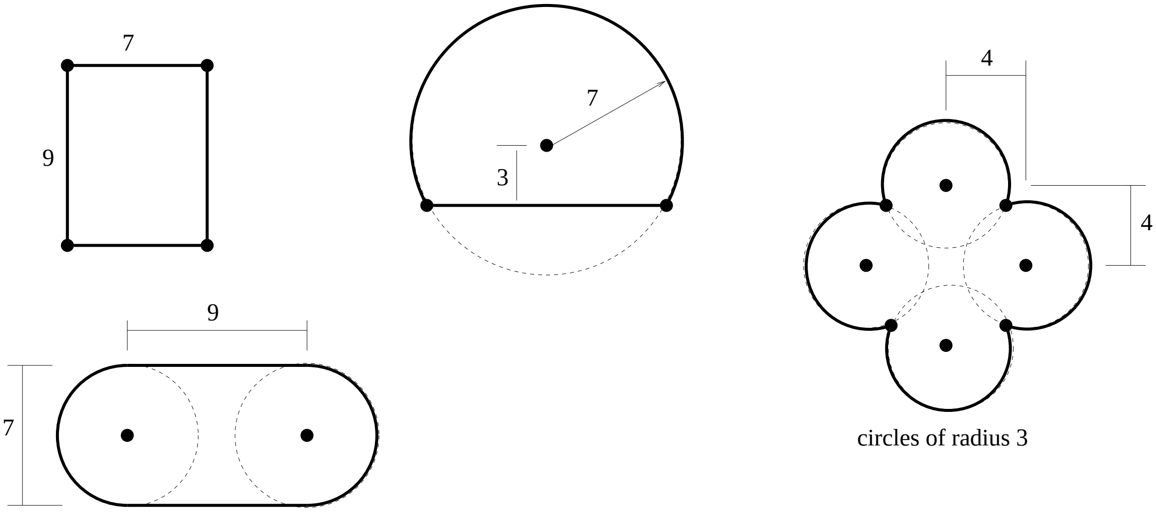
1.3 Three Simple Curves

1. This exercise emphasizes the "mechanical aspects" of circles and their equations.
   (a) Find an equation whose graph is a circle of radius 3 centered at $(-3,4)$.
   (b) Find an equation whose graph is a circle of diameter $\frac{1}{2}$ centered at the point $(3,-\frac{11}{3})$.
   (c) Find four different equations whose graphs are circles of radius 2 through (1,1).
   (d) Consider the equation $(x-1)^2+(y+1)^2=4$. Which of the following points lie on the graph of this equation: (1,1), (1,-1), (1,-3), $(1+\sqrt{3},0),(0,-1-\sqrt{3})$, (0,0).

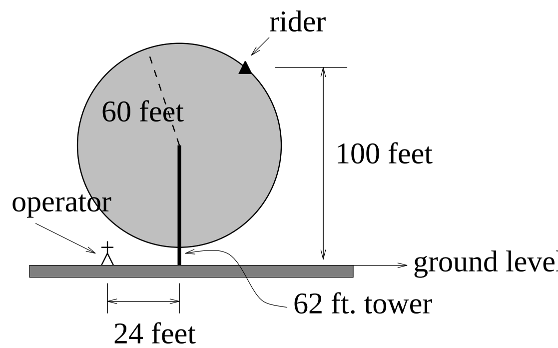
2. Find an equation whose graph is a circle passing through $P=(-10,10)$ and $Q=(50,-20)$. Determine where the circle crosses the x and y axis. Is there more than one answer? (Hint: There are a lot of answers; the easiest case is to assume the two points lie on a diameter of the circle.)

3. Draw a coordinate system containing the graphs of the equations $x=2$ and $y=-3$.
   (a) One graph is a vertical line and one graph is a horizontal line; which is which?
   (b) How many times (if any) does the graph of the equation $x^2+y^2=2$ intersect the vertical line? Find the coordinates of any intersection points.
   (c) How many times (if any) does the graph of the equation $x^2+y^2=4$ intersect the vertical line? Find the coordinates of any intersection points.
   (d) How many times (if any) does the graph of the equation $x^2+y^2=8$ intersect the vertical line? Find the coordinates of any intersection points.
   (e) How many times (if any) does the graph of the equation $x^2+y^2=3$ intersect the horizontal line? Find the coordinates of any intersection points.
   (f) How many times (if any) does the graph of the equation $x^2+y^2=4$ intersect the horizontal line? Find the coordinates of any intersection points.
   (g) How many times (if any) does the graph of the equation $x^2+y^2=8$ intersect the horizontal line? Find the coordinates of any intersection points.
   (h) Find all points of intersection between the circle of radius 6 centered at the point (-1,2) and these two lines.
   (i) Describe the region of the xy-plane containing points (x,y) that satisfy the condition: $x\geq 2$ and $y\geq -3$. Draw a picture of this region.
   (j) Describe the region of the xy-plane containing points (x,y) that satisfy the condition: $x\geq 2$ and $y\leq -3$. Draw a picture of this region.
   (k) Describe the region of the xy-plane containing points (x,y) that satisfy the condition: $x\leq 2$ and $y\geq -3$. Draw a picture of this region.
   (l) Describe the region of the xy-plane containing points (x,y) that satisfy the condition: $x\leq 2$ and $y\leq -3$. Draw a picture of this region.

4. For each of the given figures, find equation(s) that model the perimeter. Also describe the coordinates of any of the points indicated by a solid "dot".



5. An amusement park Ferris Wheel has a radius of 60 feet. The center of the wheel is mounted on a tower 62 feet above the ground (see picture). For these questions, the wheel is not turning.



(a) Impose a coordinate system.

(b) Suppose a rider is located at the point in the picture, 100 feet above the ground. If the rider drops an ice cream cone straight down, where will it land on the ground?

(c) The ride operator is standing 24 feet to one side of the support tower on the level ground at the location in the picture. Determine the location of a rider on the Ferris Wheel so that a dropped ice cream cone lands on the operator. (Note: There are two answers.)

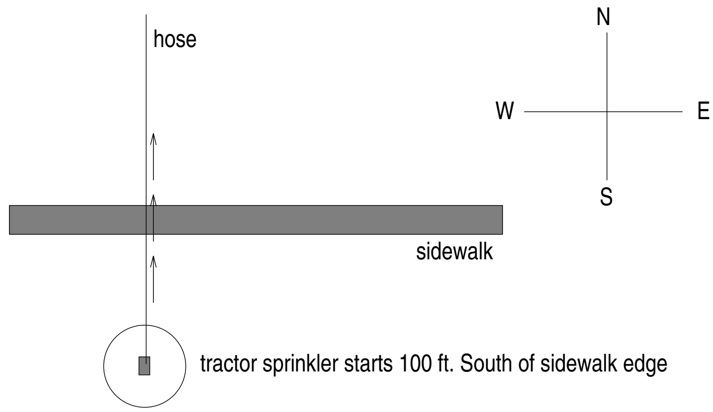
6. This is your first day on the job as a sales representative for Big-Glo Inc., designers of a full line of new halogen light fixtures for use in rural areas. This company makes custom designed fixtures which cast an illuminated area in the shape of a circular disc of a specified radius r feet. Your first customer would like to install a light 100 feet west and 20 feet south of the intersection of two perpendicular country roads. In addition, there is a gate located 150 feet north of the intersection of these two roads which needs to lie in the illuminated area of the light.

(a) Find the smallest size illuminated disc that will fit your customer's needs.

(b) Determine what portions of each road will be illuminated by the light chosen in part a.

7. Water is flowing from a major broken water main at the intersection of two streets. The resulting puddle of water is circular and the radius r of the puddle is given by the equation $r = 3t\sqrt{7}$ feet, where t represents time in minutes elapsed since the the main broke. When the main broke, a runner was located 6 miles from the intersection. The runner continues toward the intersection at the constant speed of 12 miles per hour. When and where will the runner's feet get wet?

8.* A *crawling tractor sprinkler* is located as pictured below, 100 feet South of a sidewalk. Once the water is turned on, the sprinkler waters a circular disc of radius 20 feet and moves North along the hose at the rate of $\frac{1}{2}$ inch/second. The hose is perpendicular to the 10 ft. wide sidewalk.



(a) Impose a coordinate system. Describe the initial coordinates of the sprinkler and find equations of the lines forming the North and South boundaries of the sidewalk.

(b) When will the water first strike the sidewalk?

(c) When will the water from the sprinkler fall completely North of the sidewalk.

(d) Find the total amount of time water from the sprinkler falls on the sidewalk.

(e) Sketch a picture of the situation after 33 minutes. Draw an accurate picture of the watered portion of the sidewalk.

(f) Find the area of GRASS watered after one hour.

9. Sketch the circle of radius 1 centered at (1,2) in the xy-coordinate system. Describe pictorially and via an equation (or inequality) each of these situations:

(a) The horizontal lines that do not intersect the circle.

(b) The vertical lines that do not intersect the circle.

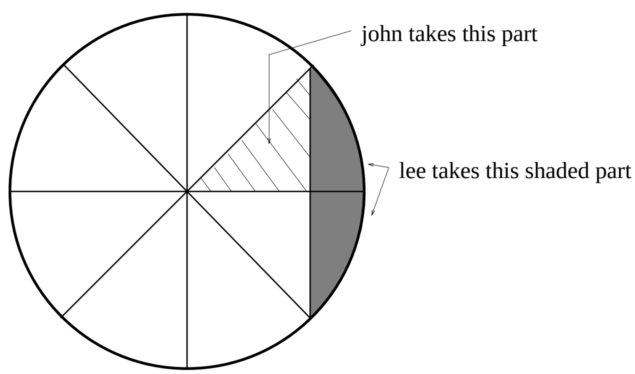
(c) The horizontal lines that intersect the circle exactly once.

(d) The vertical lines that intersect the circle exactly once.

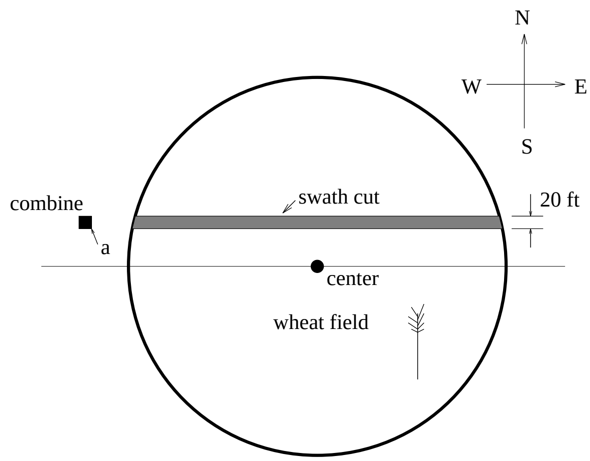
(e) The horizontal lines that intersect the circle exactly twice.

(f) The vertical lines that intersect the circle exactly twice.

10. *Aleko's Pizza* has delivered a beautiful 16 inch diameter pie to Lee's dorm room. The pie is sliced into 8 equal sized pieces, but Lee is such a non-conformist he cuts off an edge as pictured. John then takes one of the remaining triangular slices. Who has more pizza and by how much?



11. Nora spends part of her summer driving a *combine* during the wheat harvest. Assume she starts at the indicated position heading east at 10 ft/sec toward a circular wheat field of radius 200 ft. The combine cuts a swath 20 feet wide and begins when the corner of the machine labeled "a" is 60 feet north and 60 feet west of the western-most edge of the field.



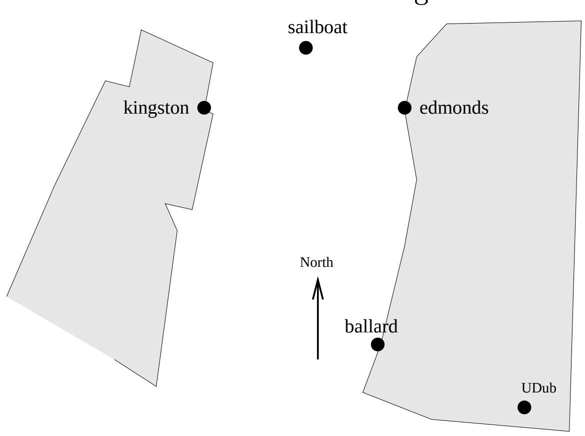
(a) When does Nora's rig first start cutting the wheat?

(b) When does Nora's rig first start cutting a swath 20 feet wide?

(c) During this pass across the field, how much time is wheat being cut?

(d) Estimate the area of the swath cut during this pass across the field.

12. Erik's disabled sailboat is floating stationary 3 miles East and 2 mile North of Kingston. A Ferry leaves Kingston heading toward Edmonds at 12 mph. After 20 minutes the ferry turns heading due South. Ballard is 8 miles South and 1 mile West of Edmonds. Impose coordinates with Ballard the origin. Edmonds is 6 miles due East of Kingston.



(a) Find the equations for the lines along which the ferry is moving and draw in these lines.

(b) The sailboat has a radar scope that will detect any object within 3 miles of the sailboat. Looking down from above, as in the picture, the radar region looks like a circular disk. The *boundary* is the "edge" or circle around this disc, the *interior* is the inside of the disk, and the *exterior* is everything outside of the disk (i.e. outside of the circle). Give a mathematical (equation) description of the boundary, interior and exterior of the radar zone. Sketch an accurate picture of the radar zone by determining where the line connecting Kingston and Edmonds would cross the radar zone.

(c) When does the ferry enter the radar zone?

(d) How would you determine where and when the ferry exits the radar zone?

(e) How long does the Ferry spend inside the radar zone?

13. Suppose you change the coordinate system in the previous problem to have origin at Kingston. Which (if any) of your answers will change?

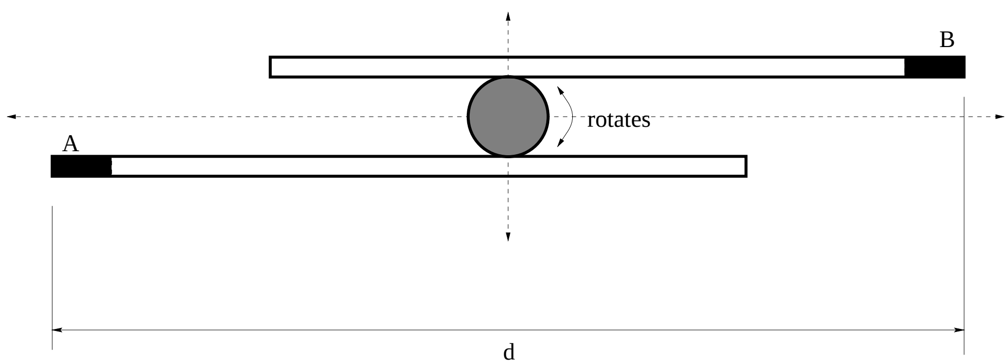
14. Because of modern sprinkler-irrigation systems, farmers often plant crops in fields that are circular in shape (as you will know if you've ever flown over eastern Washington). Farmer Jones has a circular field whose radius is 50 yards and whose center is 70 yards east and 40 yards north of his house.

    (a) Using the house as the origin of the coordinate system, write down an equation that describes the circular curve that bounds the field.

    (b) A boulder is embedded in the ground 100 yards east and 10 yards north of the house. Does the boulder lie within the field?

    (c) If Farmer Jones walks directly east from his house, how far has he gone when he gets to the edge of the field?

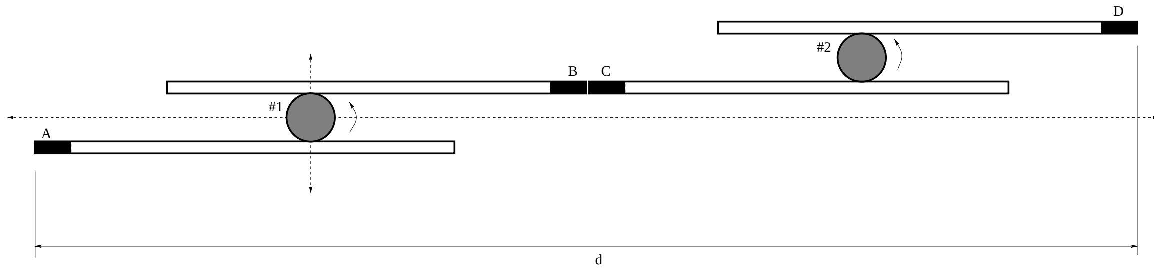
15. This problem illustrates the basic mechanism behind muscle movement. For this problem you will need the following items: two unsharpened pencils of the same length, one screwtop from bottle of pop, a piece of paper on which you have drawn a coordinate system. Arrange these items as pictured below (this is a view looking down on the table). The center of the screwtop is located at the origin of the coordinate system and we have labeled the two eraser-ends as "A" and "B".



    (a) Imagine that the center of the screwtop is fastened to the origin of the coordinate system. If the screwtop rotates counterclockwise, what happens to A and B? If the screwtop rotates clockwise, what happens to A and B? (You can create this situation by holding onto the ends of the two pencils, then moving your hands toward or away from each other.)

    (b) Continue to imagine that the center of the screwtop is fastened to the origin of the coordinate system. Suppose the screwtop rotates, resulting in the movement of A to the right at a rate of 4mm/sec. Assume both ends A and B start out a horizontal distance of 14 cm from the vertical axis; so, initially the horizontal distance from A to B is 28 cm. Find the pictured horizontal distance d after 1 second? 2 seconds? t seconds? At what rate is the distance d changing?

    (c) Now, borrow your neighbors two pencils plus screwtop and arrange the system pictured below, where the two ends of your neighbors system are labeled "C" and "D". Tape the two pencils where B and C touch:



Continue to imagine that the center of screwtop #1 is fastened to the origin of the coordinate system. Suppose screwtop # 1 rotates counterclockwise, resulting in the movement of A to the right at a rate of 4mm/sec. Simultaneously, screwtop #2 is rotating counterclockwise, resulting in the movement of D toward the vertical centerline of screwtop #2 at a rate of 4mm/sec. (Notice, as screwtop #2 rotates, it's center is moving to the left in this system.) At what rate is D moving toward the left in the coordinate system? At what rate is the pictured distance d changing?

16. A radio transmitter is located 200 yards east and 900 yards north of John's house. It transmits a signal that can be received only at locations less than 500 yards from the transmitter.

    (a) Taking John's house as the origin of the coordinate system, find the equation of the circle that forms the boundary of the receiving region.

    (b) If John walks directly north from his house, how far does he have to go to reach the receiving region?

    (c) If John walks directly north from his house at 3 mph, how long will he remain in the receiving region?

    (d) Beth's house is 650 yards east and 750 yards north of John's house. Can Beth receive the signal from her house?

    (e) The transmitter is replaced by a more powerful one so that the area of the receiving region is doubled. Can John now receive the signal from his house?

    (f) The transmitter is replaced by a more powerful one so that the perimeter of the receiving region is doubled. Can John now receive the signal from his house?

17. Draw the graphs of $x = -1, y = 2$ and a circle of radius 4 centered at the point (2,3) in the xy coordinate system. Let X be the point where the horizontal and vertical line intersect.

    (a) Determine where the vertical line and the horizontal line intersect the circle; i.e. find the coordinates of the intersections. Label all points of intersection in your picture.

    (b) An ant starts at the location (6,2) and moves to the left along the line $y = 2$. Assume the position of the ant after t seconds is the point $P(t) = (6 - 2t, 2)$. Plot the ant locations $P(0), P(1), P(2), P(4)$.

    (c) At the same instant, a spider starts at the location (-1,-3) and moves upward along the line $x = -1$. Assume the position of the spider after t seconds is the point $Q(t) = (-1, -3 + t)$. Plot the spider locations $Q(0), Q(1), Q(4), Q(6)$.

    (d) When will the ant exit the circular zone? When will the spider enter the circular zone?

    (e) Which bug reaches the point X first?

    (f) Use the distance formula to find the distance between the ant and the spider at time t seconds. Your answer will involve t.

(g) For times $t = 2, 3, 4$, draw a line segment connecting $P(t)$ and $Q(t)$ and compute its length; i.e. plot $P(2)$ and $Q(2)$ and connect these by a line segment, etc.