1.2 Imposing Coordinates

1. Using an xy-coordinate system, plot the points $A = (1,1), B = (\frac{3}{2},1), C = (2,1), D = (-1,-1), E = (-1,-\frac{1}{2}), F = (-1,0)$.

   (a) Use the distance formula to find:

      i. The distance from A to D.

      ii. The distance from B to E.

      iii. The distance from C to F.

   (b) For each number t, define a point $P(t)$ by the formula $P(t) = (1+t,1)$. Plot the points $P(0), P(0.5), P(1),\ P(-1), P(2)$.

   (c) Likewise, for each number t, define a point $Q(t)$ by the formula $Q(t) = (-1,-1+t)$. Plot the points $Q(0), Q(0.5), Q(1),\ Q(-1), Q(2)$.

   (d) Use the distance formula to compute the distance between $P(t)$ and $Q(t)$. Your formula will involve t.

   (e) Find a value of t so that the distance between $P(t)$ and $Q(t)$ is 5. Where are the two points located for this value of t?

2. Suppose two cars depart from a four way intersection at the same time, one heading north and the other heading west. The car heading north travels at the steady speed of 30 ft/sec and the car heading west travels at the steady speed of 58 ft/sec.

   (a) Find an expression for the distance between the two cars after t seconds.

   (b) Find the distance in miles between the two cars after 3 hours 47 minutes.

   (c) When are the two cars 1 mile apart?

3. Rework Example 1.2.2 by imposing a coordinate system with Aaron initially at the origin. Show that the collision location on the runway is the same as the one we just found (although the equations you will use to find the collision point are very different from the ones used in (1.2.2).

4. A Ferrari is heading south at a constant speed on Broadway (a north/south street) at the same time a Mercedes is heading west on Aloha Avenue (an east/west street). The Ferrari is 624 feet north of the intersection of Broadway and Aloha, at the same time that the Mercedes is 400 feet east of the intersection. Assume the Mercedes is traveling at the constant speed of 32 miles/hour. Find the speed of the Ferrari so that a collision occurs in the intersection of Broadway and Aloha.

5. Plot the following four points in the xy plane: $A = (1,2), B = (1,-1), C = (-2,-1), D = (-1,1)$; each axis will be scaled the same and the units will be feet. Form the region bounded by the line segments $\overline{AB}, \overline{BC}, \overline{CD}, \overline{DA}$. A bug begins at the point A and starts moving around the perimeter of the region in a clockwise direction 2 inches/second; i.e. the bug walks from A to B to C to D to A, etc. Also, assume the bug doesn't slow down at the corners.

   (a) Sketch the region.

   (b) Find the perimeter of the region.

   (c) How long does it take the bug to complete one trip around the region?

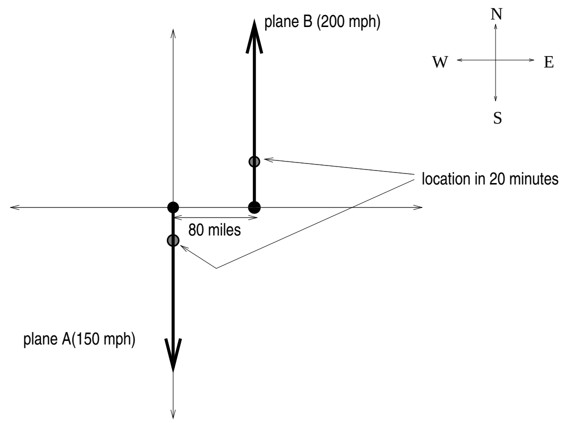
(d) Where is the bug located after 8 seconds?

(e) When will the bug first cross the x-axis?

(f) When will the bug first cross the y-axis?

(g) During the bug's first trip around the region, when is it between C and D?

6. Two planes flying opposite directions (North and South) pass each other 80 miles apart at the same altitude. The Northbound plane is flying 200 mph (miles per hour) and the Southbound plane is flying 150 mph. How far apart are the planes in 20 minutes? When are the planes 300 miles apart?



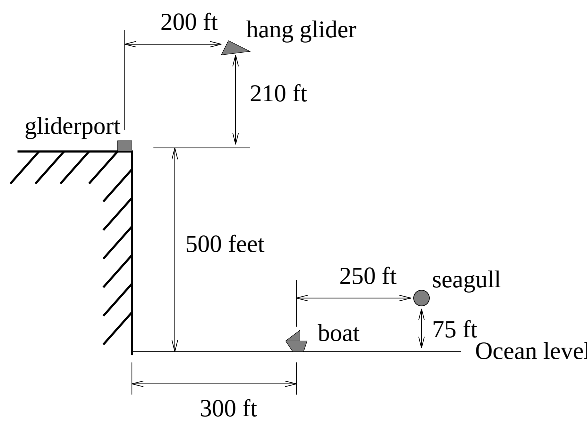
7. A hang glider launches from a gliderport in La Jolla. The launch point is located at the edge of a 500 ft. high cliff over the Pacific Ocean. Impose three different coordinate systems: one with origin at the gliderport, one with the origin at the hang glider and the third with origin at the boat location. Answer these questions for each coordinate system separately.

   (a) What are the coordinates of the hang glider?

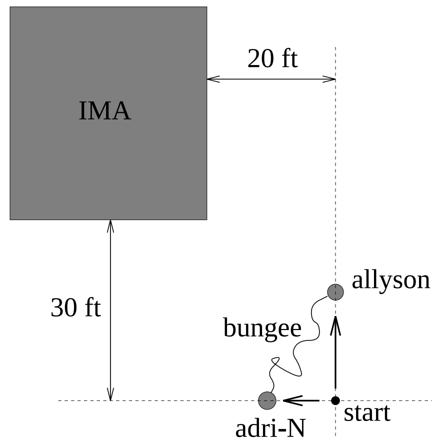
   (b) What are the coordinates of the seagull?

   (c) What are the coordinates of the boat?

   (d) What are the coordinates of the gliderport?



8. Allyson and Adrienne have decided to connect their ankles with a *bungee cord*; one end is tied to each persons ankle. The cord is 30 feet long, but can stretch up to 90 feet. They both start from the same location. Allyson moves 10 ft/sec and Adrienne moves 8 ft/sec in the directions indicated.



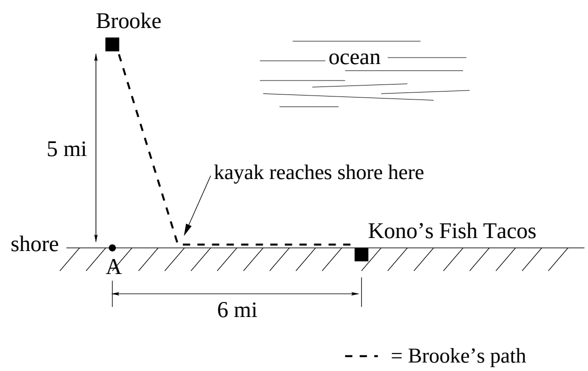
   (a) Where are the two girls located after 2 seconds?

   (b) After 2 seconds, will the slack in the bungee cord be used up?

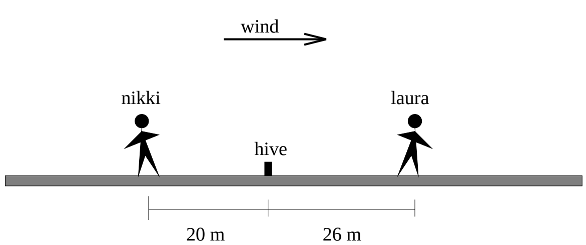
   (c) Determine when the bungee cord first becomes tight; i.e. there is no slack in the line. Where are the girls located when this occurs?

   (d) When will the bungee cord first touch the corner of the building? (Hint: Use a fact about "similar triangles".)

9.* Brooke is located 5 miles out from the nearest point A along a straight shoreline in her seakayak. Hunger strikes and she wants to make it to *Kono's* for lunch; see picture. Brooke can paddle 2 mph and walk 4 mph. If she paddles along a straight line course to the shore, find an equation that computes the the total time to reach lunch in terms of the location where Brooke beaches the boat. Determine the total time to reach Kono's if she paddles directly to either the point "A" or directly to Kono's. Do you think one of these two answers is the minimum time required for Brooke to reach lunch?



10. Nikki and Laura are standing on opposite sides of a hive of killer bees. The bees can fly 7.5 m/sec in *still air* and the wind is blowing 4.5 m/sec in the indicated direction. Notice, if the bees are flying upwind (against the wind), then the wind speed will decrease their ground speed. Two bees leave the hive; one is heading for Nikki and the other toward Laura. Assume that both women start running away from the hive at the same instant.

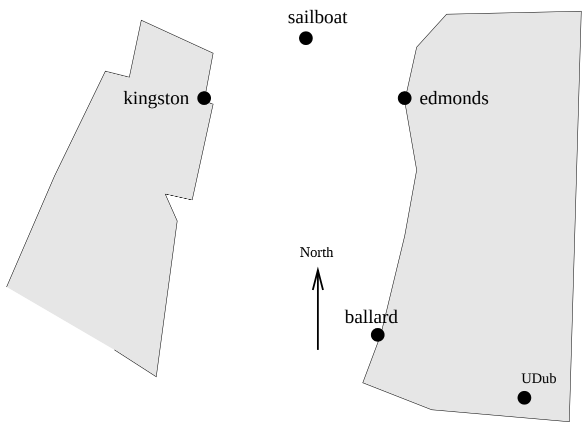


    (a) What is the ground speed of each bee?

(b) Assume each person runs 3.1 m/sec. Determine who escapes and who doesn't. If someone can't escape, determine where and when attack will occur.

(c) Determine the minimum speed each must run in order to avoid being stung. Is it realistic for Nikki to outrun the bees? How about Laura?

11. Erik's disabled sailboat is floating at a stationary location 3 miles East and 2 mile North of Kingston. A ferry leaves Kingston heading due East toward Edmonds at 12 mph. At the same time, Erik leaves the sailboat in a dinghy heading due South 10 ft/sec (hoping to intercept the ferry). Edmonds is 6 miles due East of Kingston.



(a) Compute Erik's speed in mph and Ferry speed in ft/sec.

(b) Impose a coordinate system and complete this table of data concerning locations of Erik and the ferry. Insert into the picture the locations of the ferry and Erik after 7 minutes.

	0 second	30 second	7 minutes	t hours .
Ferry coordinates				
Erik's coordinates				
Distance between them				

(c) Explain why Erik misses the ferry.

(d) After 10 minutes, a Coast Guard boat leaves Kingston heading due East at a speed of 25 ft/sec. Will the Coast Guard boat catch the ferry before it reaches Edmonds? Explain.

12. During weekends, Linda likes to go swimming in the Snohomish River. At her favorite spot, the river current is 3/4 mph and she always swims at a rate of 2 mph. While she is swimming, her necklace breaks but fortunately it floats. After it breaks, she swims upstream 40 minutes, then turns around and swims directly back to the necklace. How long does it take Linda to retrieve the necklace?

13. You want to plot the points of the form $(n\,ft., n^2\,ft.)$, $n = 0, 1, 2, \ldots, 20$. Your graph paper is 50 cm square divided into 1 cm squares. You intend to draw a coordinate system on the paper so that a one foot unit on the horizontal axis has length 2 cm on the graph paper. How must you scale the vertical axis so that all of the plotted points all fit onto your graph paper? What is the aspect ratio of this coordinate system?

14. Start with two points $M = (a, b)$ and $N = (s, t)$ in the xy-coordinate system. Let d be the distance between these two points. Answer these questions and make sure you can justify your answers:

(a) TRUE or FALSE: $d = \sqrt{(a-s)^2 + (b-t)^2}$.

(b) TRUE or FALSE: $d = \sqrt{(a-s)^2 + (t-b)^2}$.

(c) TRUE or FALSE: $d = \sqrt{(s-a)^2 + (t-b)^2}$.

(d) Suppose M is the beginning point and N is the ending point. What is Δx? What is Δy?

(e) Suppose N is the beginning point and M is the ending point. What is Δx? What is Δy?

(f) If Δx=0, what can you say about the relationship between the positions of the two points M and N ? If Δy=0, what can you say about the relationship between the positions of the two points M and N ? (Hint: Use some specific values for the coordinates and draw some pictures to see what is going on.)