

1.1 Units and Rates

- Verify that 7685.33 seconds is 2 hours 8 minutes 5.33 seconds.
 - Which is faster: 100 mph or 150 ft/s?
 - Gina's salary is 1 cent/second for a 40 hour work week. Tiare's salary is \$1400 for a 40 hour work week. Who has a higher salary?
 - Assume it takes 180 quarter credits to get a baccalaureate degree. If 1 quarter credit counts for one classroom hour of lecture each week of the quarter and you study 2.5 hours for each hour in class, how many hours must you invest to get a degree? (You may assume each quarter has 10 weeks of class and no holidays.)
- Marathon runners keep track of their speed using units of *pace* = minutes/mile.
 - Lee has a speed of 16 ft/sec; what is his pace?
 - Allyson has a pace of 6 min/mile; what is her speed?
 - Adrienne and Dave are both running a race. Adrienne has a pace of 5.7 min/mile and Dave is running 10.3 mph. Who is running faster?
- Convert each of the following sentences into "pseudo-equations". For example, suppose you start with the sentence: "The cost of the book was more than \$10 and the cost of the magazine was \$4." A first step would be these "pseudo-equations":
(Book cost) > \$10 and (Magazine cost) = \$4.
 - John's salary is \$56,000 a year and he pays no taxes.
 - John's salary is at most \$56,000 a year and he pays 15% of his salary in taxes.
 - John's salary is at least \$56,000 a year and he pays more than 28% of his salary in taxes.
 - The number of students taking Math 120 at the UW is between 1500 and 1800 each year.
 - The cost of a new red Porsche is more than three times the cost of a new Ford F-150 pickup truck.
 - Students spend between two and three hours studying outside class for each credit hour.
 - Twice the number of happy math students exceeds five times the number of happy chemistry students. However, all of the happy math and chemistry students combined is less than half the total number of cheerful biology students.
 - The difference between Cady's high and low midterm scores was 10%. Her final exam score was 97%.
- The density of lead is 11.34 g/cm³ and the density of aluminum is 2.69 g/cm³. Find the radius of lead and aluminum spheres having a mass of 50 kg.
- Which is a better deal: A 10 inch diameter pizza for \$8 or a 15 inch diameter pizza for \$16?
- The famous *theory of relativity* predicts that a lot of weird things will happen when you approach the speed of light $c = 3 \times 10^8$ m/sec. For example, here is a formula that relates the mass m_o (in kg) of an object at rest and it's mass when it is moving at a speed v :

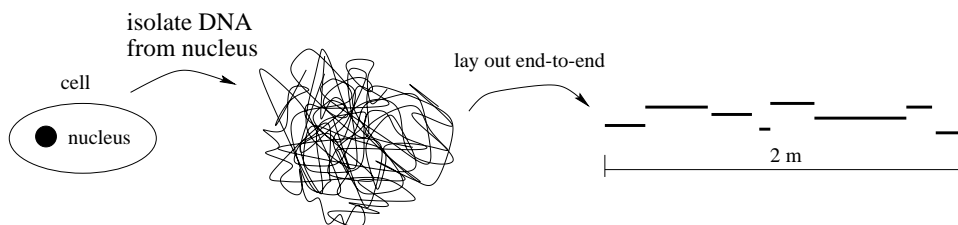
$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- (a) Suppose the object moving is Dave, who weighs $m_o = 66$ kg at rest. What is Dave's mass at 90% of the speed of light? At 99% of the speed of light? At 99.9% of the speed of light?
- (b) How fast should Dave be moving to have a mass of 500 kg?
7. This problem illustrates the principle used to make a good "squirt gun". A cylindrical tube has diameter 1 inch, then reduces to diameter d . The tube is filled with oil and piston A moves to the right 2 in/sec, as indicated. This will cause piston B to move to the right m in/sec. Assume the oil does not compress; that means the volume of the oil between the two pistons is always the same.



- (a) If the diameter of the narrow part of the tube is $\frac{1}{2}$ inch, what is the speed of piston B ?
- (b) If B moves 11 in/sec, what is the diameter of the narrow part of the tube?
8. (a) The temperature at 7:00 am is 44°F and the temperature at 10:00 am is 50°F . What are the initial time, the final time, the initial temperature and the final temperature? What is the rate of change in the temperature between 7:00 am and 10:00 am?
- (b) Assume it is 50°F at 10:00 am and the rate of change in the temperature between 10:00 am and 2:00 pm is the same as the rate in a. What is the temperature at 2:00 pm?
- (c) The temperature at 4:30 pm is 54°F and the temperature at 6:15 pm is 46°F . What are the initial time, the final time, the initial temperature and the final temperature? What is the rate of change in the temperature between 4:30 pm and 6:15 pm?
- (d) Assume it is 46°F at 6:15 pm and the rate of change in the temperature between 6:15 pm and 7:00 pm is the same as the rate in c. What is the temperature at 7:00 pm?
9. Nonresident yearly tuition at the UW in 1996 is \$9753. Suppose nonresident yearly tuition at the UW in 1998 is \$10727. What is the rate of change in yearly nonresident tuition? Assume the rate of change in yearly nonresident tuition is constant, then what would be the yearly nonresident tuition in the year 2000?
10. (a) Give two examples of a quantity that is changing at a constant rate.
- (b) Give two examples of a quantity that is not changing at a constant rate.
11. During a typical evening in Seattle, *Pagliacci* receives phone orders for pizza delivery at a constant rate: 18 orders in a typical 4 minute period. How many pies are sold in 4 hours? Assume *Pagliacci* starts taking orders at 5pm and the profit is a constant rate of: \$11 on 10 pies. When will phone order profit exceed \$1000?
12. A barrel in the shape of a circular cylinder with an open top has radius 15 inches and height 48 inches. It contains 20 gallons of old smelly water. It begins to rain and the water level rises at a constant rate of 0.35 in/hour.
- (a) After 6 hours, how much has the water level risen?
- (b) What is the total depth of the water in the barrel after 6 hours?
- (c) What is the rate at which the volume of water in the barrel is changing?

13. A water pipe mounted to the ceiling has a leak and is dripping onto the floor below, creating a circular puddle of water. The surface area of this puddle is increasing at a constant rate of $11 \text{ cm}^2/\text{hour}$.
- Find the surface area and radius of the puddle after 1 minute, 92 minutes, 5 hours, 1 day?
 - * We could also discuss the rate at which the water is leaking from the pipe. To do this, we would use units of “volume per unit time”, such as $\text{ml/hr} = \text{cm}^3/\text{hr}$. Assume that the depth of the circular puddle is always 1 mm. Is the water leaking at a constant rate? Explain your reasoning.
14. Laura has convinced you to invest in her 3 month old (90 day old) startup company, *Rad Snoboards*. She has told you that nationwide sales of their boards have held steady at a rate of 9 boards/day, seven days a week, since the first day of business.
- How many boards are sold in a typical thirty day month?
 - If you start counting from Rad Snoboards first day of business, when will total board sales for the company hit 1000 boards?
 - You are told you will receive a dividend (a check in the mail) which pays you \$1.25 for every board sold after the first 1000 in total sales (counting from Rad Snoboards first day of business). You are mailed a dividend check covering the first six weeks after you invest. How much is the check?
- 15.* Describe two different models that predict the number of ping-pong balls that will fit in our classroom. Clearly indicate the assumptions you are making in your models.¹
16. Inside a typical biological cell there will be numerous chemical reactions in progress. Cells use *enzymes* to speed up the rate of a reaction. For example, the enzyme *lactase* will break the molecule lactose (found in milk) into the two separate molecules: glucose and galactose. Suppose a cell has enough lactase to carry out this reaction 12,000 times per second. If the cell needs to break down 10^6 molecules of lactose, how long will it take? If we wanted to break down 10^6 molecules of lactose in 3 seconds, how many times as much lactase would need to be present in the cell?
17. A typical cell in the human body contains molecules of *deoxyribonucleic acid*, referred to as *DNA* for short. In the cell, this DNA is all twisted together in a tight little packet. But, imagine unwinding (straightening out) all of the DNA from a single typical cell and laying it “end-to-end”; then the sum total length will be approximately 2 meters.



Assume the human body has 10^{14} cells containing DNA. How many times would the sum total length DNA in your body wrap around the equator of the earth?

18. Ecologists estimate that 20,000 species are lost to extinction each year. Find the extinction rate per day? per hour?

¹Taken from *How to Model It*, by Starfield, Smith, Bleloch, Burgess Press 1994.

19. During the 1950's, Seattle was dumping an average of 20 million gallons of sewage into Lake Washington each day.

- (a) How much sewage went into Lake Washington in a week? In a year?
- (b) In order to illustrate the amounts involved, imagine a rectangular prism whose base is the size of a football field (100 yards \times 50 yards) with height h yards. What are the dimensions of such a rectangular prism containing the sewage dumped into Lake Washington in a single day?

(Note: Dumping into Lake Washington has stopped; now it goes into the Puget Sound.)

20. Your neighbor is having her concrete driveway broken up into pieces and replaced. You would like to use the broken pieces to lay a garden path in your back yard. Assume the pieces are on average 4 inches thick. The garden path is 30 inches wide and 64 feet long.

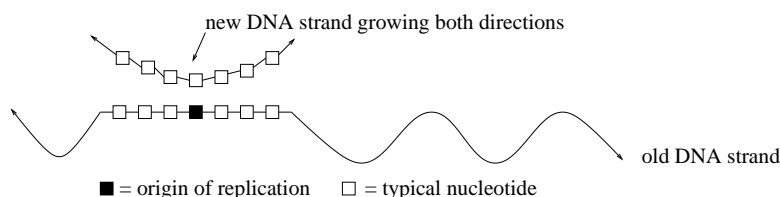
- (a) Estimate how many cubic yards of your neighbors broken concrete is needed. Make clear any assumption(s) you are making to arrive at this estimate.
- (b) After you lay the path, you find that only 10% of your neighbors broken material was needed. If new concrete costs \$120 per cubic yard, estimate the amount your neighbor will spend on new concrete.

21. *Big Ed's* pizza delivery advertizes a one-day special: All pizza's are double the usual diameter for double the usual cost. For example, if a 12 inch pie usually costs \$8, then a 24 inch pie costs \$16 during the special. Is this a good deal? Why or why not?

22. Dave has inherited an apple orchard on which 60 trees are planted. Under these conditions, each tree yields 12 bushels of apples. According to the local WSU extension agent, each time Dave removes a tree the yield per tree will go up 0.45 bushels. Let x be the number of trees in the orchard and N the yield per tree.

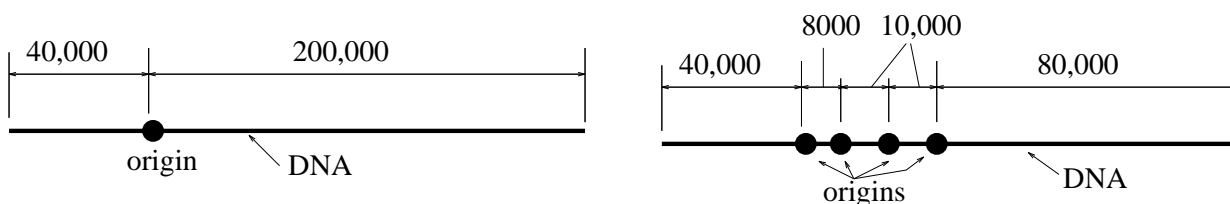
- (a) Find a formula for N in terms of the unknown x . (Hint: Make a table of data with one column representing various values of x and the other column the corresponding values of N . After you complete the first few rows of the table, you need to discover the pattern.)
- (b) What possible reason(s) might explain why the yield goes up when you remove trees?

23. One portion of the biological cell cycle involves the duplication of *DNA molecules*. DNA is built by bonding together a (very) long sequence of smaller molecules called *nucleotides*. The cartoon below illustrates the situation, where we have indicated the sense in which the nucleotides are stuck together. (There are four possible types of nucleotides and the actual order in which they are joined matters a lot. However, it will not be important for this problem.)



In order to duplicate, special proteins start to make copies of the old DNA strand in both directions. The copying process begins at an *origin of replication*; there can be one or many of these origins. The rate of duplication is given in units of nucleotides/min. If two duplicating strands meet, they join together.

Here is a picture of two different yeast DNA molecules with indicated origins of replication. We have indicated the number of nucleotides between origins. Assume the rate of duplication is a constant 4000 nucleotides/min.



- (a) In each case, discuss the extent of duplication after 1 minute and 4 minutes. Sketch a picture of each situation.
- (b) In each case, determine when duplication is completed.
24. A *prime number* is an integer greater than one such that the only positive integers that divide it evenly (i.e. with remainder zero) are itself and 1. For example, 2, 3, 5, 7, 11, 13, 17, ... are the first few prime numbers. Recently, someone discovered a prime number with 900,000 digits. If you were going to write down this number and each digit requires 1.5 mm, how long would the number be? If you could speak at the rate of 2 digits/second for 8 hours each day, how long would it take you to recite the number? (The exact number is $p = 2^{2976221} - 1$, which your calculator will not like very much!)
25. Congress is debating a proposed law to reduce tax rates. If the current tax rate is $r\%$, then the proposed rate after x years is given by this formula:

$$\frac{r}{1 + \frac{1}{1 + \frac{1}{x}}}$$

Rewrite this formula as a simple fraction. Use your formula to calculate the new tax rate after 1, 2, 5 and 20 years. Congress claims that this law would ultimately cut peoples tax rates by 75%. Do you believe this claim?