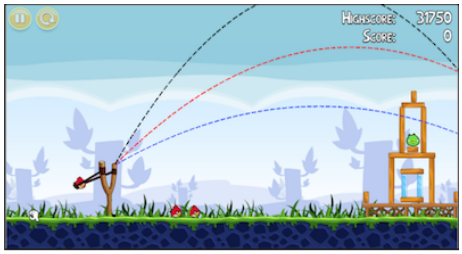
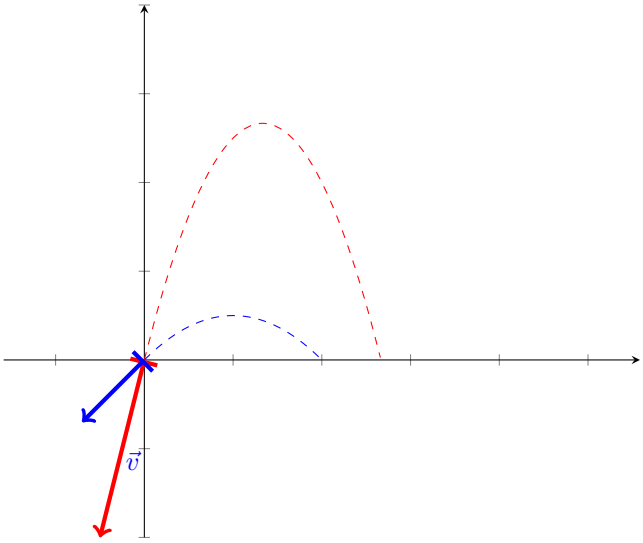
1. You are developing a prototype Angry Birds software to help high school students understand vectors and linear algebra. You would like students to input a vector, $\vec{v} = (v_1, v_2)$, and have the output, $f(\vec{v}) = (a_1, a_2, a_3)$ be the coefficients of a quadratic equation that corresponds to the direction and scale of the vector:

$$y = a_1x^2 + a_2x + a_3.$$





(a) Suppose that your vector is based at the origin. What does this mean for the value a_3?

(b) Note that $\vec{v}$ points the **opposite direction** of the flight of the bird. It is required that birds land in front of their launch point at (0,0). What constraints does this impose on v_1 and v_2? Write them down, or explain why there are none.

(c) Write the equation that corresponds to Span$\{\vec{v}\}$.

(d) You want the slope of the tangent line to the quadratic and the line from part (c) to agree at 0. Write the corresponding linear equation.

(e) Suppose you decide to set $a_1 = v_1 + v_2$. Explain why this is a reasonable choice.

(f) Write a matrix corresponding to the system of equations for a_1, a_2, a_3.

(g) A student asks whether you can now find $\vec{v}$ so that the bird passes through two specified points $(a, b), (c, d)$. Write the system of equations that $\vec{v}$ must satisfy for this to happen.

(h) Is it possible if instead three arbtrary points are specified? Why or why not?

(i) You input your answer from part (g) into a computer and obtain the system in echelon form:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{b}{a^2} + \frac{\frac{bc^2}{a^2} - d}{a\left(c - \frac{c^2}{a}\right)} \\ 0 & 1 & 0 & -\frac{\frac{bc^2}{a^2} - d}{c - \frac{c^2}{a}} \\ 0 & 0 & 1 & 0 \end{array}\right)$$

What values of (a, b, c) are not acceptable. Explain why. Don't just say because the matrix equations don't make sense.

(j) Let

$$g(a,b) = \frac{b}{a^2} + \frac{\frac{bc^2}{a^2} - d}{a\left(c - \frac{c^2}{a}\right)} \quad \text{and} \quad h(a,b) = -\frac{\frac{bc^2}{a^2} - d}{c - \frac{c^2}{a}}.$$

Given $(a, b), (c, d)$ write a **linear** system of equations for v_1 and v_2 in terms of $h(a, b)$ and $g(a, b)$.

(k) What is the solution to this equation?

2. A video game designer is rectifying the dynamics of two different rooms in a video game. She needs to know if $\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\}$ and $\left\{ \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \right\}$ have the same span.

   (a) Call (x, y, z) a point in the span of a set of vectors. Write down two augmented matrices that correspond to the span of the two sets of vectors above.

   (b) Say you row reduce these matrices. What must happen for them to have the same span?

3. Byeol is a programmer at Google and she is given a piece of code which she is told starts with two vectors $\{\vec{u}, \vec{v}\}$ (that she does not know) and ultimately outputs $\{\vec{u} + \vec{v}, \vec{u} - \vec{v}\}$.

   (a) Is it possible to determine the values of $\vec{u}$ and $\vec{v}$ from the output? Explain.

   (b) Byeol is told that $\vec{u}$ and $\vec{v}$ are linearly independent. She needs to know that the output also is. Show that $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are also linearly independent