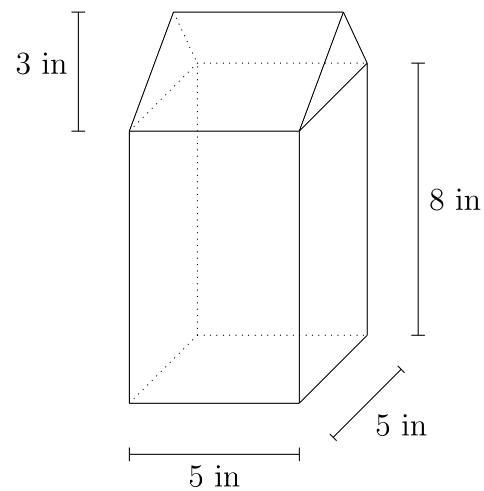
Worksheet for Week 7: Related Rates

This worksheet guides you through some more challenging problems about related rates.



A milk carton is shaped like a tall box with a triangular prism on top. The sides of the top section are isosceles triangles. This particular milk carton has a 5 inch $\times$ 5 inch square base, and is 11 inches tall. (See the picture.)

Suppose you're filling the carton with liquid, at a rate of 10 inches^3 per minute. In this problem, you'll figure out the rate of change of the height of the liquid in the carton, at the instant when the carton holds 220 inches^3 of liquid.

1. (a) Let y be the height of the liquid in the carton. Then y is a function of time, because y is changing as more liquid pours in. Suppose $y \le 8$, so that the liquid is all in the rectangular part of the carton. Find a formula for the total volume of liquid in the carton.

   (b) Now suppose $8 \le y \le 11$, so that some liquid is in the triangular part of the carton. Find a formula for the total volume of liquid in the carton, in terms of y. You might want to break up the volume into two pieces: the volume below the 8-inch line and the volume above the 8-inch line.

(c) Next, suppose that 220 in^3 of liquid is in the carton. How high is the liquid level? That is, what is y?

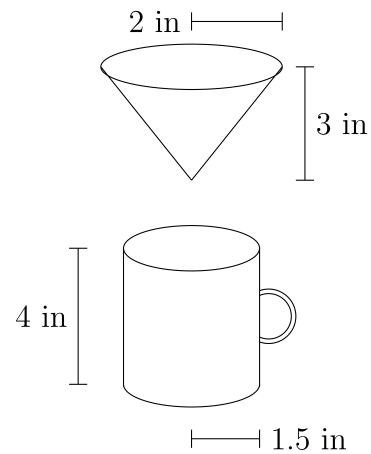
(d) What is $\frac{dy}{dt}$ when 220 in^3 of liquid is in the carton?

(e) When y reaches 8 inches, does $\frac{dy}{dt}$ increase, decrease, or stay the same? You can answer this question by using the formulas you found on Page 1, or by looking at the picture. How do you know your answer is correct?

2. Now suppose a funnel is positioned over a coffee mug.

   The funnel is a cone 3 inches high with a base radius of 2 inches, and the coffee mug is 4 inches tall with a 1.5-inch radius. See the picture at right.

   Suppose the funnel is initially **full** of coffee. Then it starts to drip down into the mug at a constant rate of 0.1 inches^3 per second.



   (a) Let y be the height of the coffee in the funnel at any given time, so that just before the dripping starts we have $y = 3$. Since the coffee is draining out of the funnel, $\frac{dy}{dt}$ will be negative. What (approximately) will be the value of y when $\frac{dy}{dt}$ is at its smallest (closest to zero)?

   (b) What (approximately) will y be when $\frac{dy}{dt}$ is biggest (farthest from zero)?

   (c) How much coffee is in the funnel at the very beginning?

   (d) How much coffee is in the funnel **and mug** at some time t?

(e) Find a formula, depending on the height y of coffee in the funnel, for the volume of coffee in the funnel.

(f) Now suppose the coffee **mug** is one-third full of coffee. How fast is the height of coffee in the funnel changing? In other words, what is $\frac{dy}{dt}$ at that instant?