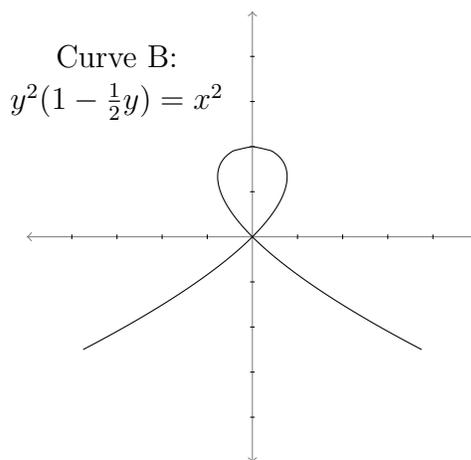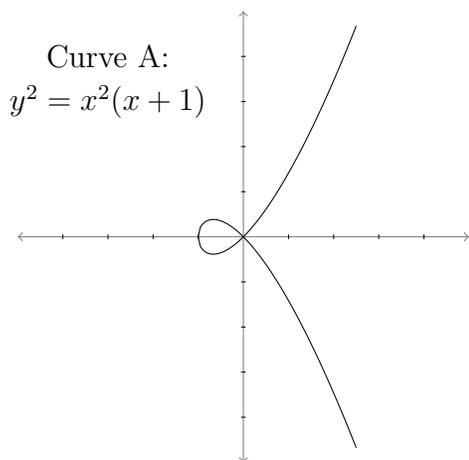# Worksheet for Week 6: Implicit Differentiation

In this worksheet, you'll use parametrization to deal with curves that have more than one tangent line at a point. Then you'll use implicit differentiation to relate two derivative functions, and solve for one using given information about the other.

Curve A:
$$y^2 = x^2(x+1)$$

Curve B:
$$y^2(1 - \tfrac{1}{2}y) = x^2$$

1. (a) Use impicit differentiation to find all the points in Curve A with a horizontal tangent line. (Looking at the graph, how many such points should there be?)
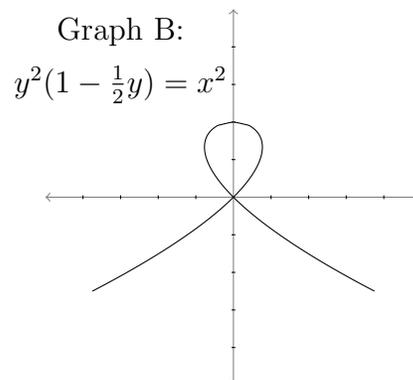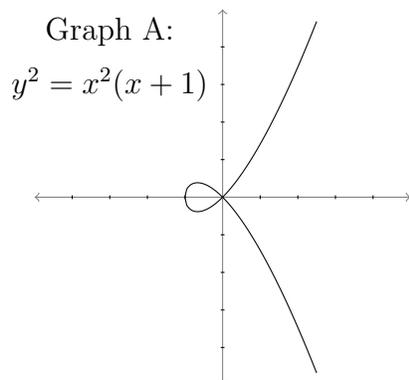
   (b) What about Curve B?

   (c) Try to find $\frac{dy}{dx}$ at the point $(0,0)$ on both graphs. What goes wrong?

2. In this problem you'll look at the curves from Page 1 in a different way.

Suppose a cat is chasing a ball around on the floor, and its position is described by the parametric equations

$$(x(t), y(t)) = (t^2 - 1, t - t^3).$$

Graph A:　　　　　　　　　　　　　　　　　Graph B:

$y^2 = x^2(x + 1)$　　　　　　　　　　　　$y^2(1 - \frac{1}{2}y) = x^2$

(a) The cat is following one of the paths from the previous page (reprinted above). Which path does the cat follow? Circle this curve. How do you know it's the right one?

(b) Draw an arrow on the circled graph above, to indicate in which direction the cat is running.

(c) At which time(s) t does the cat run through the point $(0,0)$?

(d) Remember that it wasn't possible to find $\frac{dy}{dx}$ at $(0,0)$ using the method on Page 1. But now that the graph has been parametrized, you can do it. What are the tangent line(s) to the parametrized curve $(x(t), y(t))$ at $(0,0)$?

3. This next question is a new type of problem that you can solve now that you know about implicit differentiation. Suppose a snowball is rolling down a hill, and its radius r is growing at a rate of 1 inch per minute. The volume V of the snowball grows more quickly as the snowball gets bigger. In this question, you'll find the rate of change of the volume, $\frac{dV}{dt}$, at the instant when the radius r is 6 inches.

   (a) First, apply geometry to the situation. Can you think of an equation that relates the variables r and V to each other?

   (b) Now the variables V and r change as time changes, so we can think of them as functions of t. Differentiate the equation you came up with in part (a) with respect to t.

   (c) What is the rate of change of the radius? Use this to simplify your equation from part (b).

   (d) What is the rate of change of V when the radius of the snowball is 6 inches?