MATH 581G: HOMEWORK ASSIGNMENT # 5

DUE MONDAY, DECEMBER 11

Complete the remaining 5 problems from HW 4 and the following.

(1) Let K be a field and let v be a discrete valuation on K.
   (a) Prove that v extends uniquely to the completion K_v.
   (b) Let $\mathcal{O}_{K,v}$ be the valuation ring of K and let $\mathcal{O}_v$ be the valuation ring of K_v. Let $\mathfrak{p}$ be the maximal ideal of $\mathcal{O}_{K,v}$ and let $\mathfrak{p}_v$ be the maximal ideal of $\mathcal{O}_v$. Prove that
   $$\mathcal{O}_{K,v}/\mathfrak{p}^n \cong \mathcal{O}_v/\mathfrak{p}_v^n$$
   for all non negative integers n.

(2) Let K be a field and let v be a valuation on K. Prove that v extends uniquely to any purely inseparable extension L/K.

(3) (a) Show that if $p \neq 2$, then $\mathbb{Q}_p^\times/\mathbb{Q}_p^{\times 2} \cong (\mathbb{Z}/2\mathbb{Z})^2$ with representatives $\{1, p, a, ap\}$, where $a \in \mathbb{Z}_p^\times$ is such that $a \bmod p \notin \mathbb{F}_p^{\times 2}$.
   (b) Show that $\mathbb{Q}_2/\mathbb{Q}_2^\times \cong (\mathbb{Z}/2\mathbb{Z})^3$ with representatives $\{1, 3, 5, 7, 2, 6, 10, 14\}$.

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