1. Problem 1.10 from the textbook.

The following two problems are about symmetries of an equilateral triangle. Let I denote the identity symmetry ("do nothing"), let F denote the reflection across the vertical axis, and let R denote rotation clockwise by $120°$. Recall from class that $R^3 = I$ and $F^2 = I$.

2. Prove that the following relations are equivalent.
   (a) $FR = R^2F$
   (b) $RF = FR^2$
   (c) $RFR = F$
   (d) $FRF = R^2$
   (e) $FRFR = I$
   (f) $FR^2FR^2 = I$

3. Simplify the following expressions, i.e., write them as I, R, R^2, F, FR, or FR^2. Show your work.
   (a) $FRFR^2F^4RFR$
   (b) $R^2FRFR^2F^5RFR$
   (c) $FR^2FR^2FRFR^2$

4. Let X be a set and let $*$ be an associative binary operation. Prove that for any positive integer n and for any $x_1, x_2, \ldots x_n \in X$, the expression

$$x_1 * x_2 * \cdots * x_n$$

   is unambiguous, i.e., that regardless how we insert parentheses into the expression to indicate the order in which the operations should be carried out, we always get the same result. *Suggestion:* Use induction to show that any insertion of parentheses gives the same answer as

$$x_1 * (x_2 * (x_3 * \cdots * (x_{n-1} * x_n))).$$

5. Problem 1.8 from the textbook.

6. Let X be a set. Let $A_1, A_2, \ldots, A_n$ be subsets of X. By the two previous problems,

$$A_1 \triangle A_2 \triangle \ldots \triangle A_n$$

   is well-defined. Prove that for any positive integer n and any subsets $A_1, A_2, \ldots, A_n$, an element $x \in X$ is contained in

$$A_1 \triangle A_2 \triangle \ldots \triangle A_n$$

   if and only if $x \in A_j$ for an odd number of elements $j \in \{1, 2, \ldots, n\}$.

7. Let X be a set. Is there a subset $I \subset X$ such that for all $A \in P(X)$, $I \triangle A = A \triangle I = A$?