MATH 581G: HOMEWORK ASSIGNMENT # 2

DUE MONDAY, OCTOBER 30

Problems 2.3, 2.4, 2.5, 2.8, 3.1 from Osserman's notes and the following.

(1) Prove the following Proposition.

Proposition 0.1. *Let R be a Dedekind domain and $S \subset R$ be a multiplicative subset. Then $S^{-1}R$ is a Dedekind domain and the map*

$$\mathfrak{a} \mapsto S^{-1}\mathfrak{a}$$

gives a surjective homomorphism from the group of fractional ideals of R to the group of fractional ideals of $S^{-1}R$. The kernel of this homomorphism is generated by the integral ideals of R that meet S.

(2) **Stickelberger's criterion**. Let $K/\mathbb{Q}$ be a number field with $[K : \mathbb{Q}] = n$. Fix algebraic integers $\alpha_1, \ldots, \alpha_n$ and let $\mathrm{Hom}_{\mathbb{Q}}(K, \overline{\mathbb{Q}}) = \{\sigma_1, \ldots, \sigma_n\}$. The determinant $\det([\sigma_i(\alpha_j)]_{i,j})$ is a sum of $n!$ terms, one for each permutation of $\{1, \ldots, n\}$. Let P denote the sum of terms corresponding to the even permutations and N denote the sum of terms corresponding to the odd permutations. Show that $P + N$ and PN are integers, and that $\mathrm{disc}_{K/\mathbb{Q}}(\alpha_1, \ldots, \alpha_n) = (P + N)^2 - 4PN$. Conclude that $\mathrm{disc}_{K/\mathbb{Q}}(\alpha_1, \ldots, \alpha_n)$ is 0 or 1 modulo 4.

(3) Let $\mathfrak{a} \neq 0$ be an ideal in a Dedekind domain $\mathcal{O}$.
   (a) Prove that $\mathcal{O}/\mathfrak{a}$ is a principal ideal domain.
   (b) Prove that $\mathfrak{a}$ can be generated by two elements.

University of Washington, Department of Mathematics, Box 354350, Seattle, WA 98195, USA

E-mail address: bviray@math.washington.edu

URL: http://math.washington.edu/~bviray