Storyboarding

What do you want the audience to remember about your theorem (or talk)?

What do you want the audience to remember about your theorem (or talk)?

What is the main question?

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What is the main question?

What is your hook?

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In this paper, we study sporadic points and, more generally, isolated[4] points of arbitrary degree, focusing particularly on such points corresponding to non-CM elliptic curves. We prove that non-CM non-cuspidal sporadic, respectively isolated, points on $X_1(n)$ map to sporadic, respectively isolated, points on $X_1(d)$, for d some bounded divisor of n.

Theorem 1.1. *Fix a non-CM elliptic curve E over k, and let m be an integer divisible by $2, 3$ and all primes ℓ where the ℓ-adic Galois representation of E is not surjective. Let $M = M(E, m)$ be the level of the m-adic Galois representation of E and let f denote the natural map $X_1(n) \to X_1(\gcd(n, M))$. If $x \in X_1(n)$ is sporadic, respectively isolated, with $j(x) = j(E)$, then $f(x) \in X_1(\gcd(n, M))$ is sporadic, respectively isolated.*

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The presence of isolated points high in the tower can be detected low in the tower.

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Before this point, I need to convey

- why do we care about isolated points?
- why is this new?

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What is the main question?

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What is the main question?

What is the geometric reason that allows a curve to have infinitely many degree d points?

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What is the main question?

Before this point, I need to convey

- why do we expect there to be some geometric reason?
- why do we care about having infinitely many degree d points?

What is the geometric reason that allows a curve to have infinitely many degree d points?

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What is the hook?

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What is the hook? (For a NT seminar)

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Faltings's theorem:
Prototypical example of geometry controls arithmetic

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Would NOT work for a general audience talk!!!

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(For a NT seminar)

Faltings's theorem: example of geometry controls arithmetic

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Faltings's theorem: example of geometry controls arithmetic

What is the geometric reason that allows a curve to have infinitely many degree d points? i.e., what is the analog of Faltings

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Faltings's theorem: example of geometry controls arithmetic

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Faltings's theorem: example of geometry controls arithmetic

↙

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⟶

Isolated points are the degree d points "without a good reason to exist" (mysterious)

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Faltings's theorem: example of geometry controls arithmetic

What is the geometric reason that allows a curve to have infinitely many degree d points? i.e., what is the analog of Faltings

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What are the isolated points on modular curves?

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Faltings's theorem: example of geometry controls arithmetic

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What are the isolated points on modular curves?

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What are modular curves and why do we care?

What are the isolated points on modular curves?

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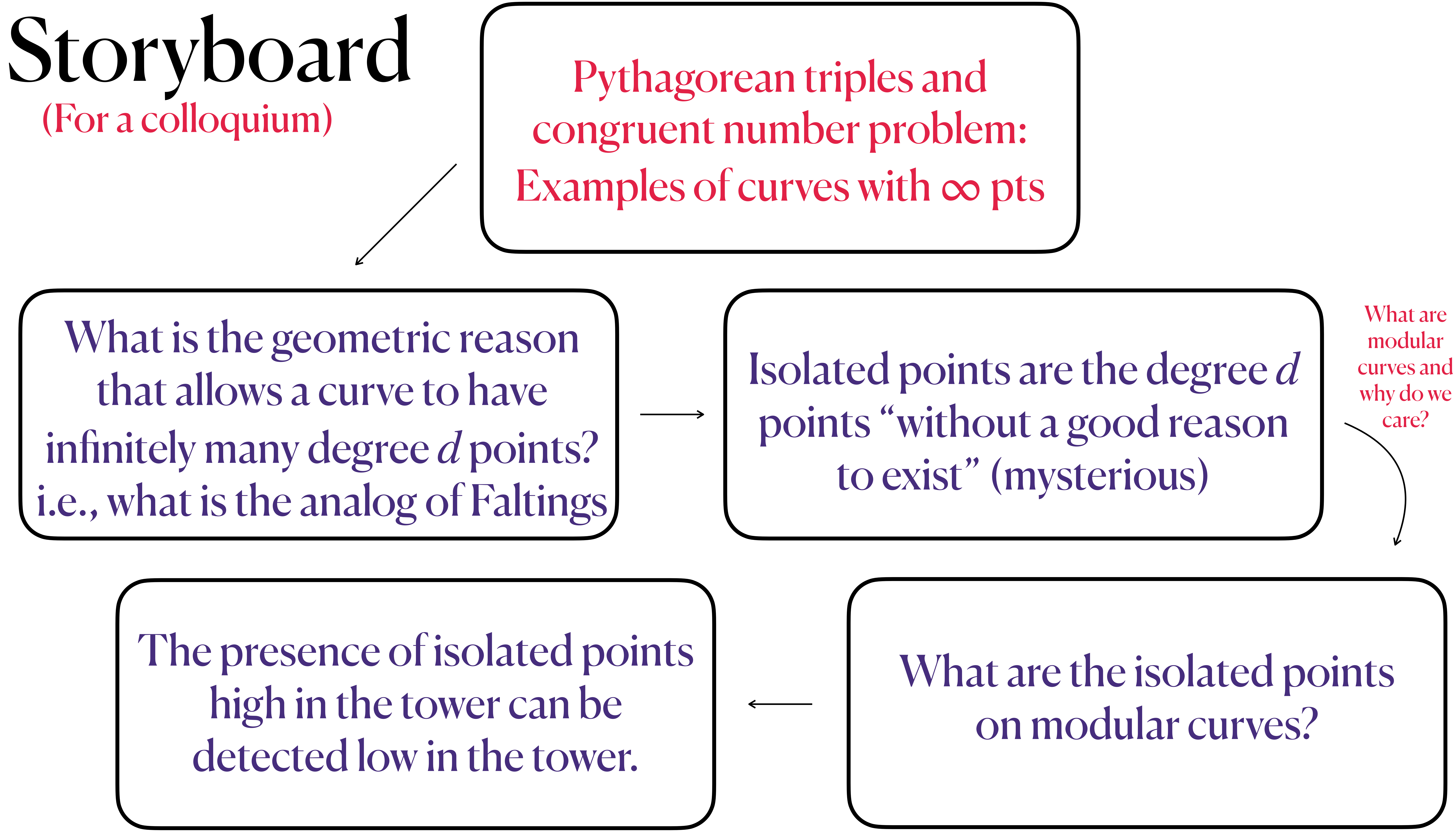
Storyboard
(For a colloquium)
Pythagorean triples and congruent number problem: Examples of curves with ∞ pts
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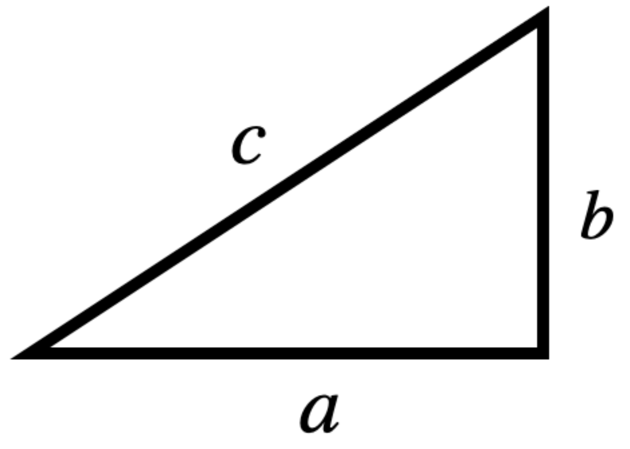
Pythagorean triples and congruent number problem: Examples of curves with ∞ pts

Pythagorean triples and congruent number problem: Examples of curves with ∞ pts

Is there a right triangle with every side length rational?



Fix a positive integer n.

Is there a right triangle with every side length rational ***and*** with area n?

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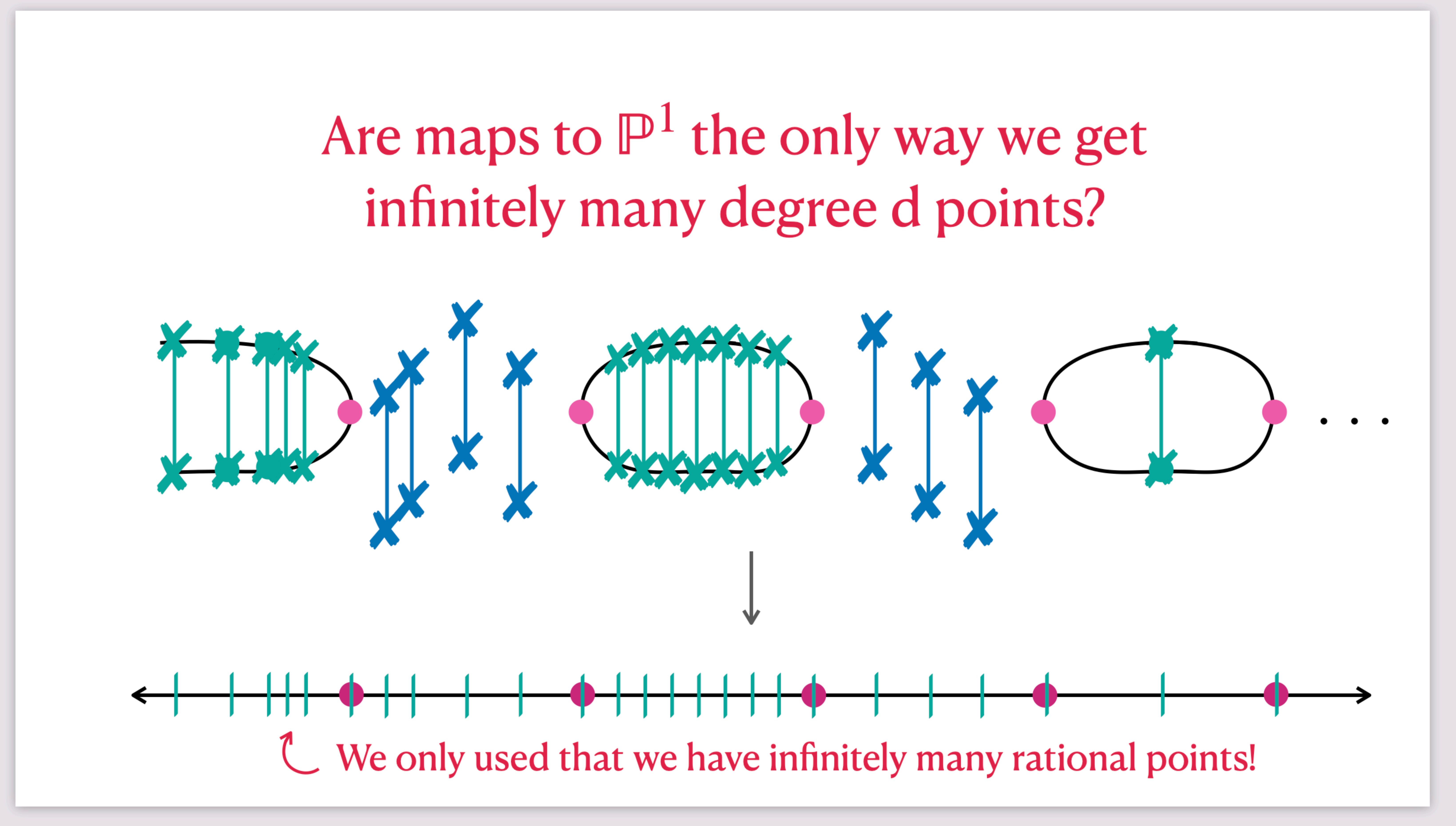

Isolated points are the degree d points "without a good reason to exist" (mysterious)

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Parametrized vs. Isolated

Parametrized points are "better understood". They cast shadows that can be detected by geometric techniques.

Isolated points 🤷‍♀️

- There are only finitely many of them.
- Guess: "Most" curves don't have any. 🤷‍♀️

What are the isolated points on modular curves?

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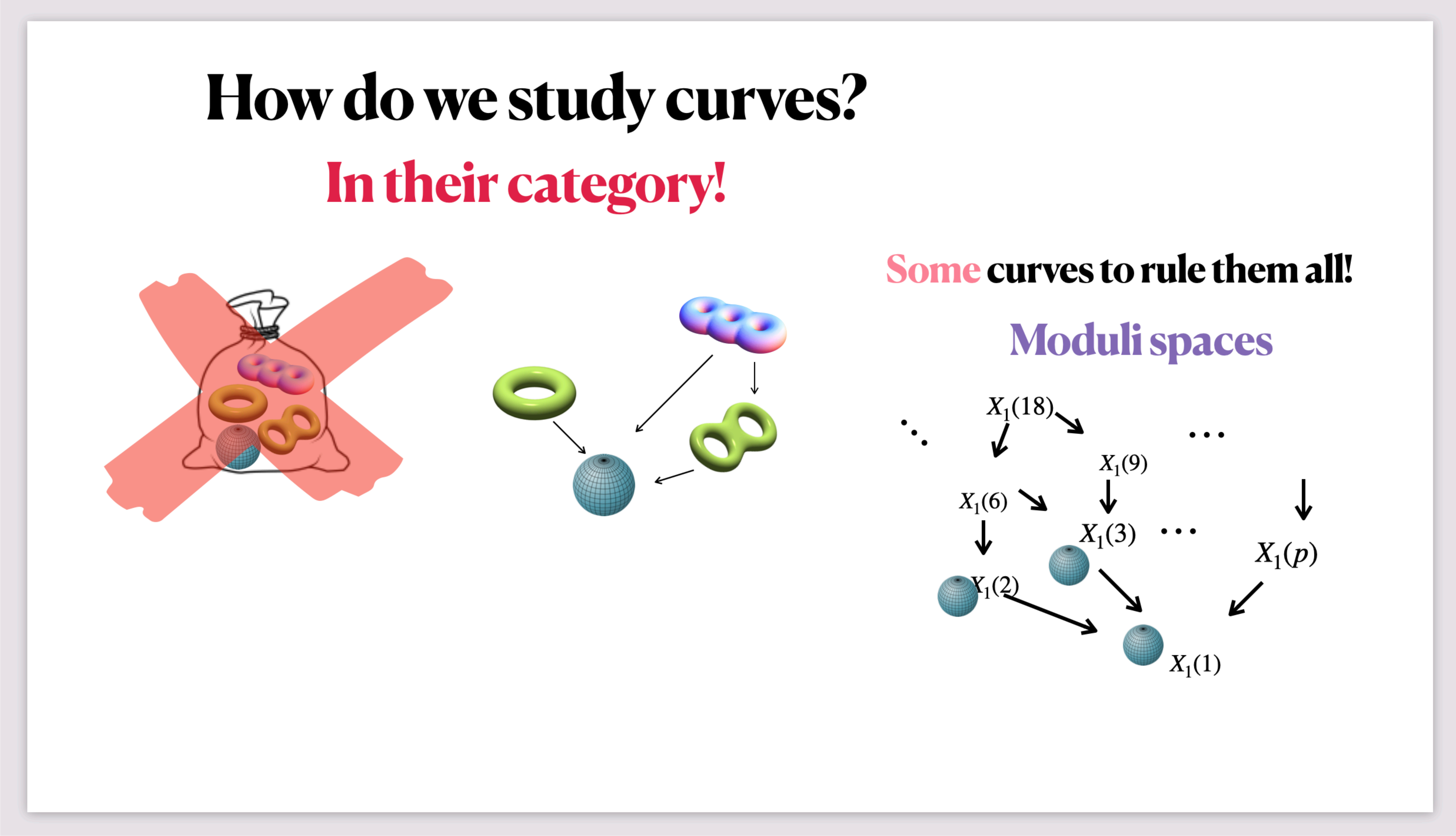

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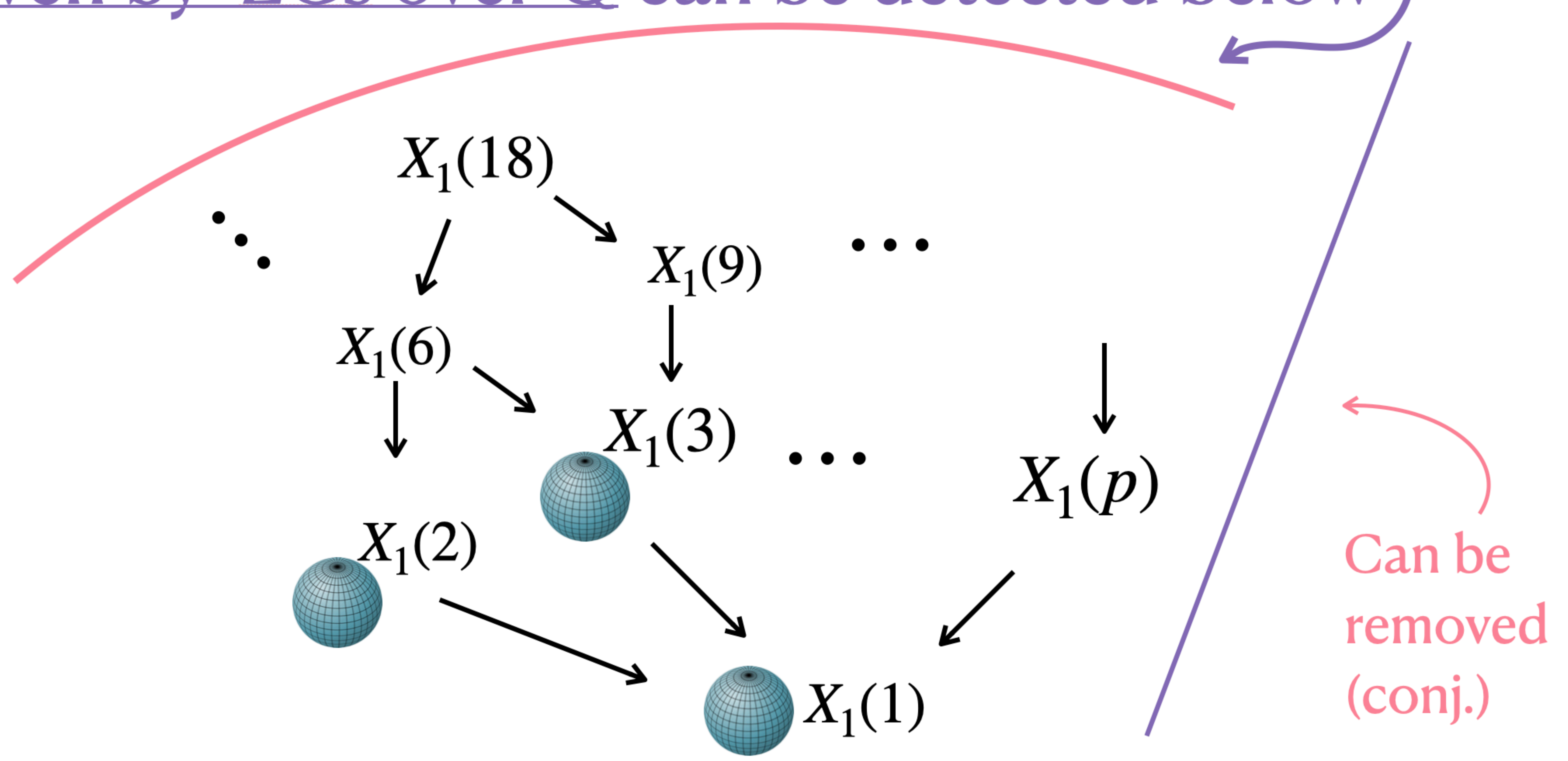
Theorem (BELOV): Non-CM isolated points given by ECs over $\mathbb{Q}$ can be detected below



Storyboard, Example 2

(For a colloquium)

The study of rational points is HARD

Quadratic reciprocity says that *p*-adic info. for all primes encodes some positivity info.; surprising!

QR means that the Sun-Tzu remainder theorem fails for infinitely many primes.

The reciprocity (Brauer-Manin) obstruction has a geometric interpretation on quartic del Pezzos

Storyboard, Example 2

(For a colloquium)

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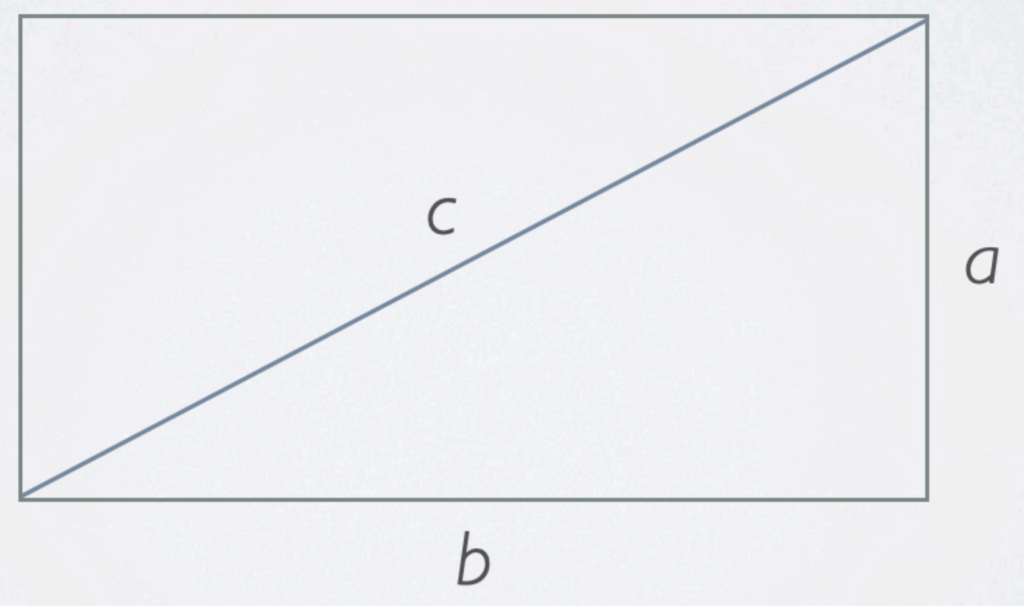
MOTIVATING EXAMPLE

Does there exist an n-dimensional box such that the distance between *any* two vertices is rational?

$n = 2$: **YES!** Pythagorean triples



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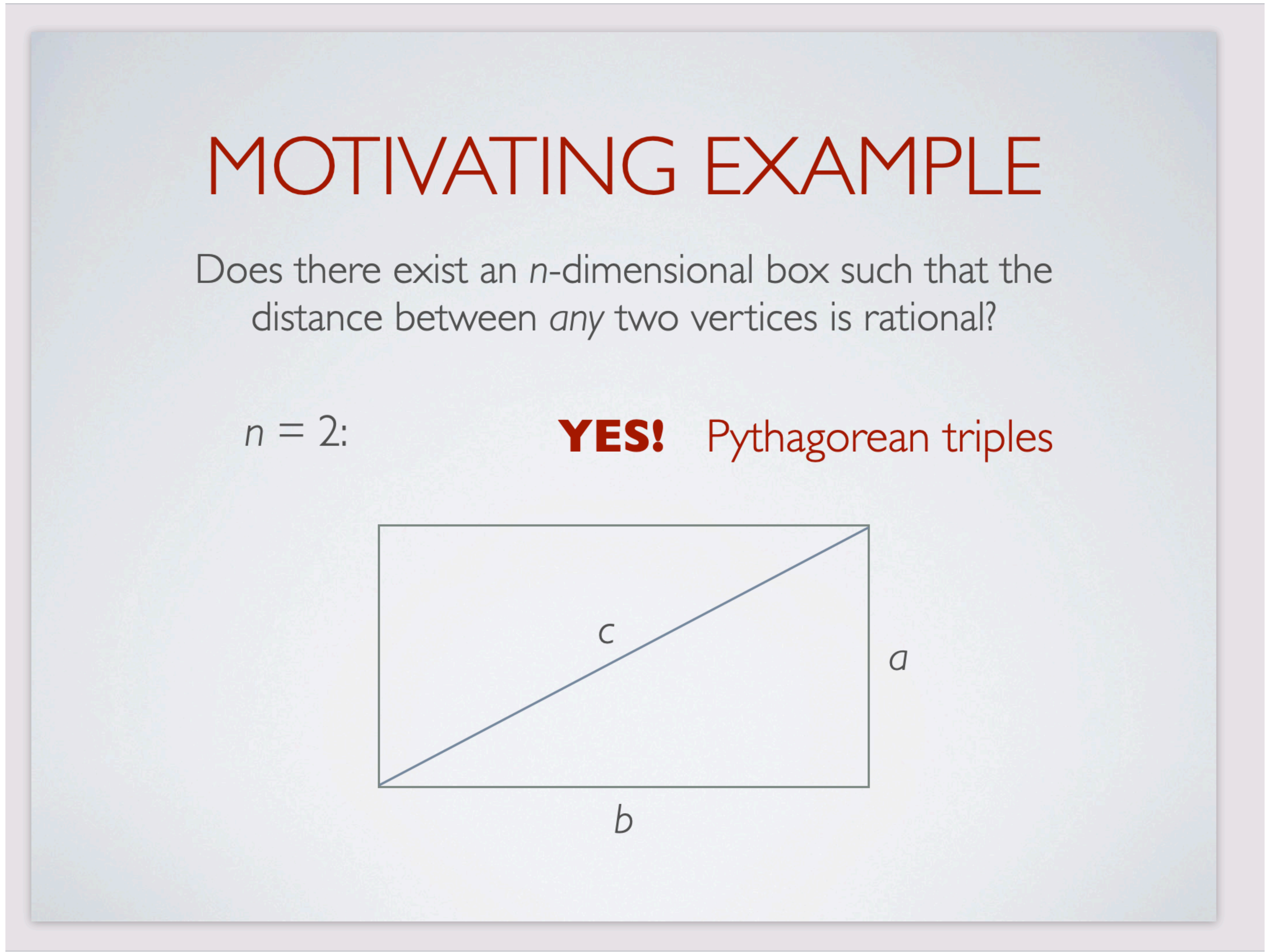

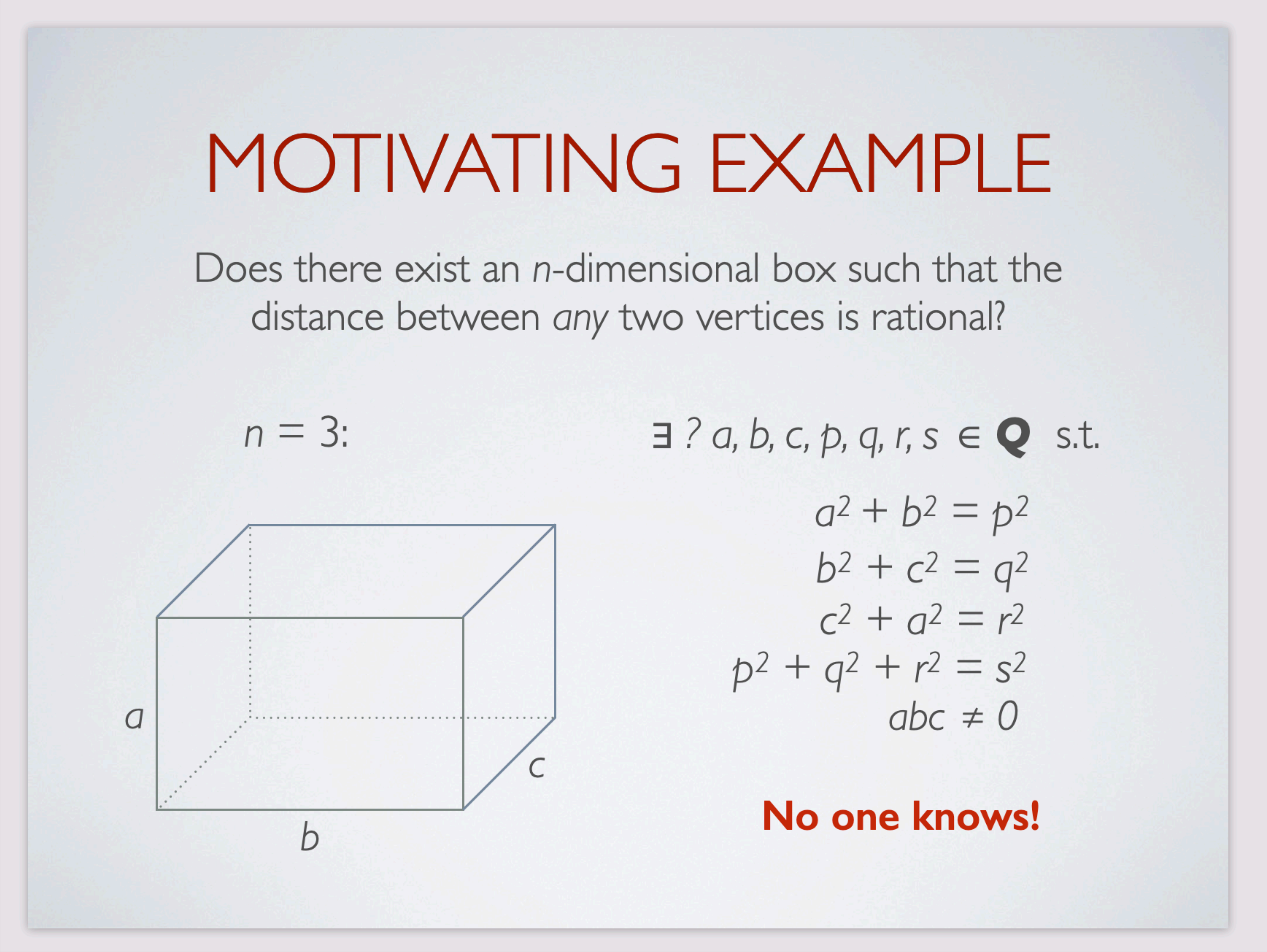

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QUADRATIC RECIPROCITY

An alternate formulation

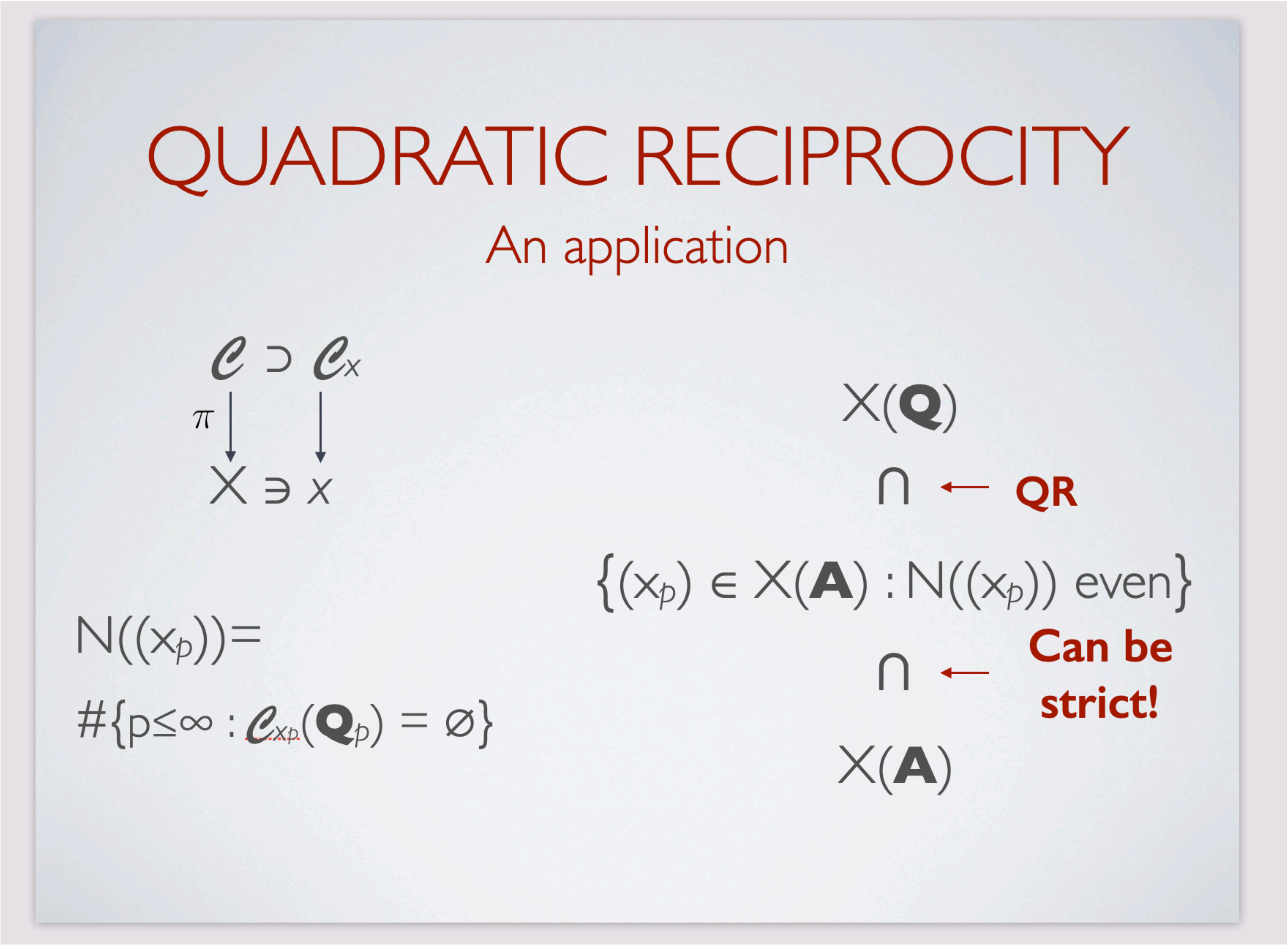
Fix $a, b \in \mathbf{Q}^\times$ and consider $C_{a,b} : ax^2 + by^2 = z^2$.

Then

$$\#\{\ p \leq \infty\ :\ C_{a,b}(\mathbf{Q}_p) = \varnothing\ \}$$

is **even**!

QR means that the Sun-Tzu remainder theorem fails for infinitely many primes.



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