Communication and knowledge transfer

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How do you read a math paper?

How do you read a novel or news article?

Digesting a math paper often requires independent work

Mathematics is commonly explained and recorded in symbolic and concrete forms that are easy to communicate, rather than in conceptual forms that are easy to understand once communicated. Translation in the direction conceptual -> concrete and symbolic is much easier than translation in the reverse direction, and symbolic forms often replaces the conceptual forms of understanding.

~ Bill Thurston, Mathoverflow.net

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There is no time for the audience to do independent work during a talk!

What can we learn from documentaries, popular math and science books, newspaper articles, etc.?

1. **Point of View:** What is the main point of the story and what is the perspective of the author?
2. **A Dramatic Question**: A key question that keeps the viewer's attention and will be answered by the end of the story.
3. **Intellectual Connection**: Serious issues that come alive in a powerful way and connects the audience to the story.
4. **The Gift of Your Voice**: A way to personalize the story to help the audience understand the context.
5. **Economy**: Using just enough content to tell the story without overloading the viewer.
6. **Pacing**: The rhythm of the story and how slowly or quickly it progresses
7. **Characters:** Who are the main characters and how does the audience remember, differentiate them, and relate to them?

Adapted from *Elements of Digital Storytelling, The Center for Digital Storytelling*

Characters

In a traditional story:

- People or beings

- Guide audience through a story
- Plot and theme are revealed through character interactions

Interpretation for a math talk:

- Mathematical objects and maybe lemmas/propositions/theorems
- Guide audience through motivation/proofs
- Insights and novelty of a result are character revelations/development

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Lessons

Don’t introduce too much notation at once.
Emphasize which lemmas/propositions/theorems are the main players
Emphasize distinguishing characteristics so audience can remember and differentiate

Examples of this aspect from traditional stories? What is the purpose of this aspect in traditional storytelling?

What is the analogue or translation of this to a math talk?

What lessons can we learn about the structure of a math talk from this aspect of storytelling?

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1. Point of view

2. A dramatic question

3. Intellectual connection

4. The gift of your voice

5. Economy

6. Pacing

Every good talk tells a story. [The talk begins] by setting the scene and introducing the characters. Then an obstacle [appears]: a problem to be solved, a theorem to be proved. Next comes the rising action, where we struggle with the problem, and may need to take a detour and/or call on friends (other parts of mathematics) to help. Then a victorious climax in which our problem is solved, followed by the falling action or denouement (consequences, applications, future work). Depending on the talk, the denouement can be omitted -- ending with a mic drop is not a bad way to close, especially in a short talk.

~ Drew Sutherland

I think afresh while preparing a talk: Why would other people care about the problem? What are the historical origins? What are the connections to other areas? What are the truly key ideas in the proof? How can I strip them of technicalities so that people can remember them intuitively?

~ Lillian Pierce

Tell a story. At the beginning of your talk you should mention some topic, problem, or theorem that everyone will agree is interesting. Your goal is to present a question to which the audience will want to know the answer; this provides suspense. You then satisfy the audience by providing the answer, or providing a partial answer, or providing the answer to an analogous question.

~ Jordan Ellenberg

Phrasing your theorem or definitions

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Given a prime p and an integer a, we define the Legendre symbol

$$\left(\frac{a}{p}\right) = \begin{cases} 1 & \text{if } a \in \mathbb{F}_p^{\times 2} \\ -1 & \text{if } a \notin \mathbb{F}_p^{\times 2} \end{cases}$$

Quadratic reciprocity

Let p, q be odd primes. Then $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{(p-1)(q-1)}{4}}$.

Quadratic reciprocity, v2

Let $f(x) = ax^2 + bx + c$, and let $D = \text{disc}(f) = b^2 - 4ac$.

Then the factorization of $f(x)$ modulo a prime p depends only on the congruence of p modulo D.

Quadratic reciprocity, v3

Let a,b be nonzero rational numbers. Consider the equation $C_{a,b} : ax^2 + by^2 = z^2$. Then

$$\#\{p \leq \infty : C_{a,b} \text{ has a nontrivial solution over } \mathbb{Q}_p\}$$

is even.

Examples of definitions:

- p-adic integers
- Compactness
- Genus of a Riemann surface

Storyboarding your talk

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Creating a storyboard for a math talk

1. Start with four 8.5”x11” sheets of blank paper, numbered 1 through 4. These are the panels of your storyboard.
2. Take panel **#3** and create a tagline or sketch for your main theorem. This should convey what you want your **audience’s take-home message** to be. It will probably NOT be what you write on the board! **[Due Monday April 6]**
3. Now that you know what you want to convey to your audience, think about the background, motivation, or context that the audience needs to appreciate your theorem. The taglines or sketches for this content go on panels **#1** and **2**.
4. Now think about applications of your result or important aspects of the proof. The taglines or sketches for this content go on panel **#4**.