QUALIFYING EXAM SYLLABUS

BIANCA VIRAY

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1. Major Topic: Algebraic Number Theory (Algebra)

Number Fields: Dedekind domains, rings of integers, norm, trace, and discriminant, finiteness of the class group, Dirichlet Unit Theorem.
Extensions of Number Fields: Decomposition and inertia groups, Hilbert's Ramification Theory, prime factorization in extension fields, Frobenius automorphism.
Valuations: Completions, valuations, extensions of valuations, Hensel's lemma, local fields, Henselian fields, unramified, tamely ramified extensions, approximation theorem.
Class Field Theory: Statements of local and global class field theory, Artin Reciprocity, Chebotarev Density Theorem (statements only), adeles and ideles.

References:

Chapter 1 §1-10, Chapter 2 §1-8 of

[1] Neukirch, J. *Algebraic Number Theory.* Grund. der Math. Wiss. 322 Springer, Berlin, 1999.

Chapters VI & VII of

[2] Cassels & Fröhlich, A. (Eds.), *Algebraic Number Theory.* Academic Press, London, 1967.

2. Major Topic: Algebraic Geometry (Algebra)

Sheaves and Schemes: Affine, reduced, irreducible, regular, fibred product.
Morphisms: finite, finite type, immersions, separated, proper, projective.
Sheaves of Modules: Quasi-coherent, coherent, twisting sheaf.
Divisors: Weil divisors, Cartier divisors, Picard group.
Morphisms to Projective Space: Closed immersions, ample, very ample.
Cohomology: Cohomology of sheaves, cohomology of projective space, Cech cohomology.
Curves: Riemann-Roch.

Reference: Chapter 2 §1-7, Chapter 3 §1-5, Chapter 4 §1 of

[1] Hartshorne, R. *Algebraic Geometry* Springer, New York, 1977.

3. Minor Topic: Complex Analysis (Classical Analysis)

Holomorphic and meromorphic functions: Taylor and Laurent series, linear fractional transformations, Liouville's theorem, Rouche's theorem.
Complex Integration: Cauchy's theorem, Cauchy's integral formula, Morera's theorem, residue theorem.
Product Developments: Weierstrass products, Hadamard's Theorem.

Reference: Chapters 1-4, Chapter 5, §1-3 of [1] Ahlfors, L. *Complex Analysis*, 3rd edition, McGraw-Hill, New York, 1979.