Topics:

- Linear systems
- Echelon form and reduced echelon form
- Applications of linear systems
- Vectors, vector equations
- Span, Linear independence
- Linear transformations
- Inverses
- Transposes
- Subspaces
- Basis, dimension
- Row space, column space, null space
- Rank, nullity
- Determinant
- Change of basis
- Dot products
- Orthogonality
- Projection
- Least squares solution
Determine whether the following sentence makes sense as a mathematical statement. You may assume that $A$ is a matrix, $T$ is a linear transformation, and that $v_1, \ldots, v_k$ are vectors in $\mathbb{R}^n$. You may also assume that all matrices and vectors are the correct sizes to be able to multiply.

1. $A$ has rank $m$.
2. $A$ is linearly independent.
3. $A$ has nullity 1.
4. $A$ spans $\mathbb{R}^n$.
5. $A$ is one-to-one.
6. $A$ is onto.
7. $A$ has at most one solution.
8. $A$ has at least one solution.
9. $A$ has no solutions.
10. $A$ has rank $n$.
11. $T$ has rank $m$.
12. $T$ is linearly independent.
13. $T$ has nullity 1.
14. $T$ spans $\mathbb{R}^n$.
15. $T$ is one-to-one.
16. $T$ is onto.
17. $T$ has at most one solution.
18. $T$ has at least one solution.
19. $T$ has no solutions.
20. $T$ has rank $n$.
21. $Ax = b$ has rank $m$.
22. $Ax = b$ is linearly independent.
23. $Ax = b$ has nullity 1.
24. $Ax = b$ spans $\mathbb{R}^n$. 
25. $Ax = b$ is one-to-one.

26. $Ax = b$ is onto.

27. $Ax = b$ has at most one solution.

28. $Ax = b$ has at least one solution.

29. $Ax = b$ has no solutions.

30. $Ax = b$ has rank $n$.

31. $\{v_1, \ldots, v_k\}$ has rank $m$.

32. $\{v_1, \ldots, v_k\}$ is linearly independent.

33. $\{v_1, \ldots, v_k\}$ has nullity 1.

34. $\{v_1, \ldots, v_k\}$ spans $\mathbb{R}^n$.

35. $\{v_1, \ldots, v_k\}$ is one-to-one.

36. $\{v_1, \ldots, v_k\}$ is onto.

37. $\{v_1, \ldots, v_k\}$ has at most one solution.

38. $\{v_1, \ldots, v_k\}$ has at least one solution.

39. $\{v_1, \ldots, v_k\}$ has no solutions.

40. $\{v_1, \ldots, v_k\}$ has rank $n$. 
Let $A$ be an $n \times m$ matrix and let $T : \mathbb{R}^m \to \mathbb{R}^n$ be the linear transformation $T(x) = Ax$. Group the following statements into sets of equivalent statements.

1. $T$ is one-to-one.
2. $T$ is onto.
3. $\ker(T) = \{0\}$.
4. $\text{Range}(T) = \mathbb{R}^n$.
5. The equation $Ax = 0$ has exactly one solution.
6. For all $b \in \mathbb{R}^n$, the equation $Ax = b$ has at most one solution.
7. For all $b \in \mathbb{R}^n$, the equation $Ax = b$ has at least one solution.
8. The columns of $A$ are linearly independent.
9. The columns of $A$ span $\mathbb{R}^n$.
10. $\text{Col}(A)$ has dimension $m$.
11. $\text{Col}(A)$ has dimension $n$.
12. $\text{Row}(A)$ has dimension $m$.
13. $\text{Row}(A)$ has dimension $n$.
14. $A$ has rank $m$.
15. $A$ has rank $n$.
16. $A$ has nullity 0.
17. $\text{Null}(A) = \{0\}$.
18. $A$ has nullity $m - n$.
19. If $T(u) = T(v)$ then $u = v$.
20. For all $b \in \mathbb{R}^n$, $T(x) = b$ has at least one solution.
21. For all $b \in \mathbb{R}^n$, $T(x) = b$ has at most one solution.
1. Let $T: \mathbb{R}^m \to \mathbb{R}^m$ be a function. To check if $T$ is a linear transformation, I need to...

2. Let $S$ be a subset of $\mathbb{R}^n$. To check if $S$ is a subspace, I need to....

3. Let $v_1, v_2, \ldots, v_k \in \mathbb{R}^n$ be a set of vectors. To check if these vectors form an orthogonal set, I need to...

4. Let $A$ be an $n \times n$ matrix, let $\lambda \in \mathbb{R}$ be a scalar, and let $v \in \mathbb{R}^n$ be a vector. To check if $v$ is an eigenvector of $A$ with eigenvalue $\lambda$, I need to....

5. Let $A$ be an $n \times n$ matrix. To determine the eigenvalues of $A$, I need to...

6. Let $A$ be an $n \times n$ matrix. To compute $A^{-1}$ (if it exists), I need to....

Prove each of the following statements. (Remember “or” is inclusive! If the prerequisites for a class are 308 or 307, and you have taken both, you can still take the class!)

1. Let $T_1: \mathbb{R}^m \to \mathbb{R}^k$ and $T_2: \mathbb{R}^k \to \mathbb{R}^n$ be linear transformations.
   (a) If $T_2 \circ T_1$ is onto, then $T_1$ is onto.
   (b) If $T_2 \circ T_1$ is one-to-one, then $T_1$ is one-to-one.
   (c) If $T_2 \circ T_1$ is not onto, then $T_1$ is not onto or $T_2$ is not onto.
   (d) If $T_2 \circ T_1$ is not one-to-one, then $T_1$ is not one-to-one or $T_2$ is not one-to-one.

2. If $\{v_1, v_2, v_3\}$ are linearly dependent vectors of $\mathbb{R}^n$, then $\{v_1 + v_2, v_2 + v_3, v_1 + v_3\}$ are linearly dependent vectors of $\mathbb{R}^n$.

3. If $\{v_1, v_2, v_3\}$ are linearly independent vectors of $\mathbb{R}^n$, then $\{v_1 + v_2, v_2 + v_3, v_1 + v_3\}$ are linearly independent vectors of $\mathbb{R}^n$.

4. If $\{v_1, v_2, v_3\}$ span a subspace $S$, then $\{v_1 + v_2, v_2 + v_3, v_1 + v_3\}$ span the same subspace $S$.

5. If $\{v_1 + v_2, v_2 + v_3, v_1 + v_3\}$ span a subspace $S$, then $\{v_1, v_2, v_3\}$ span the same subspace $S$. 


Match a, b, c, d to i, ii, iii, iv
For each picture, determine whether each function \((T_1, T_2, \text{ and their composition})\) is one-to-one and whether each function is onto.