Professor Constantin Zalinescu kindly informed us that Theorem A.1 part 4 is missing the hypothesis that $X$ be Banach. With this hypothesis a correct proof of this result is given below.

**Lemma** If $C$ is a non-empty closed convex subset of the Banach space $X$, then for all $y \in X$

$$\text{dist} (y \mid C) = \sup_{x \in C} \text{dist} (y \mid x + T_C(x)).$$

**Proof** By Lemma 5 in *(Weak sharp minima revisited, part II: applications to linear regularity and error bounds, Math. Programming, 104(2005)235-261)*

$$\bar{\mathcal{B}} = \text{dom}(\psi_C^\star) \subset \text{cl}(\text{Im}(\partial \psi_C)) = \text{cl} \left( \bigcup_{x \in C} N_C(x) \right).$$

On the other hand, by the Aubin-Ekeland result (Part 1. of Theorem A.1 in *(WSM:I)*), we have

$$\bigcup_{x \in C} N_C(x) \subset \text{dom}(\psi_C^\star) = \bar{\mathcal{B}}.$$

Therefore, if $\mathcal{B}$ is the closed unit ball in $X$, then

$$\mathcal{B} \cap \text{cl} \left( \bigcup_{x \in C} N_C(x) \right) = \mathcal{B} \cap \bar{\mathcal{B}}.$$

(1)

Now note that if $K \subset X^*$ is a cone, then $\mathcal{B} \cap \text{cl}(K) = \text{cl}(\mathcal{B} \cap K)$ since if $z_i \to \bar{z}$ with $\|z_i\| = 1$, then

$$\left\| \frac{z_i}{\|z_i\|} - \bar{z} \right\| \leq \frac{1}{\|z_i\|} \left( \|z_i - \bar{z}\| + \|z_i\| - \|\bar{z}\| \right).$$

Hence (1) becomes

$$\text{cl} \left( \mathcal{B} \cap \bigcup_{x \in C} N_C(x) \right) = \mathcal{B} \cap \bar{\mathcal{B}}.$$ (2)

Therefore,

$$\sup_{x \in C} \text{dist} (y \mid x + T_C(x)) = \sup_{x \in C} \psi_{\mathcal{B} \cap N_C(x)}(y - x)$$

$$= \sup \left\{ (z, y) - (z, x) \mid x \in C, z \in \mathcal{B} \cap N_C(x) \right\}$$

$$= \sup \left\{ (z, y) - \psi_C^\star(z) \mid x \in C, z \in \mathcal{B} \cap N_C(x) \right\}$$

$$= \sup \left\{ (z, y) - \psi_C^\star(z) \mid z \in \mathcal{B} \cap \bigcup_{x \in C} N_C(x) \right\}$$

$$= \sup \left\{ (z, y) - \psi_C^\star(z) \mid z \in \text{cl} \left( \mathcal{B} \cap \bigcup_{x \in C} N_C(x) \right) \right\}$$

$$= \sup \left\{ (z, y) - \psi_C^\star(z) \mid z \in \mathcal{B} \cap \bar{\mathcal{B}} \right\}$$

$$= \text{dist} (y \mid C).$$