Defining Variables and Writing Equations

Example
In ten years, Jack will be twice as old as he was 5 years ago. How old is Jack today?

Solution Strategy
In problems of this type there are three tasks to perform.
1) Identify the unknowns and give them variable names.
2) Translate the problem into algebraic expressions of the variables.
3) Solve the equations for the values of the variables that make the equations true.

Label the Unknowns:
In this problem we are explicitly requested to determine Jack's age. His age is not now known to us so we label his age as the variable J:

$$\text{Jack's age} \;=\; J \;.$$

Translate:
How should one divide this sentence in half so that each half lies on opposing sides of an equals sign? One way to see how to do this is to rewrite the sentence so that it is more easily translated. While rewriting the sentence, plug in the phrase "Jack's age" wherever appropriate. For example, we can rewrite the sentence as

In ten years **Jack's age** *will be twice what* **Jack's age** *was 5 years ago.*

In this sentence we see that the "=" replaces the verb "will be", so we have

$$[\text{in ten years } \textit{Jack's age}] \;=\; \left[\begin{array}{c}\text{twice what } \textit{Jack's age}\\ \text{was 5 years ago}\end{array}\right] \;.$$

We now translate the phrase *"in ten years Jack's age"* into an algebraic expression using the definition **J= Jack's age**:

$$[\text{in ten years } \textit{Jack's age}] \;=\; (J+10) \;.$$

In the phrase *twice what* **Jack's age** *was 5 years ago*, the word *twice* means multiplication by two, and

$$[\text{what } \textbf{Jack's age} \text{ was 5 years ago}] \;=\; (J-5) \;.$$

Putting this all together we get

$$(J+10) = [\text{in ten years } \textit{Jack's age}] \;=\; \left[\begin{array}{c}\text{twice what } \textit{Jack's age}\\ \text{was 5 years ago}\end{array}\right] = 2(J-5) \;,$$

or equivalently,

$$J + 10 = 2(J-5) \;.$$

Solving the equation $J + 10 = 2(J - 5)$.

J+10	=	2(J-5)	
J+10	=	2J-10	(expand the right hand side)
+10		+10	(move all numbers to the left by adding 10 to both sides)
J+20	=	2J	
-J		-J	(move all variables to the right by subtracting J from both sides)
20	=	J	

Now check the solution by plugging $J = 20$ back into the equation $J + 10 = 2(J - 5)$.

$$J + 10 = 20 + 10 = 30 \quad \text{and} \quad 2(J - 5) = 2(20 - 5) = 2 \times 15 = 30 \ .$$

This checks out so we have successfully solved this problem.

Try the following problems using the approach described above.

1. In twenty years, Mark will be three times as old as he was 10 years ago. How old is Mark today?

2. When Alex was 5 years old he was one third of what his age will be in five years. How old is Alex?

3. Two years less than three times Mary's current age is the same as one year less than two times her current age. How old is Mary?

4. Four years from now three times Julie's age will be her age twenty four years from now. How old is Julie?

5. Sixty one minus Joe's age is five times Joe's age last year. How old is Joe?

6. Carl went to the store and bought three dozen eggs for \$1.20 per dozen. On his way home he met Tracy. He gave Tracy back the \$1.50 he borrowed from her last week. He now has exactly half the money he had before going to the store. How much money does he have now?

Some solutions
1. Mark is 25, 3. Mary is 1, 5. Joe is 11, 6. Carl now has \$5.10