Translating Sentences of a Single Variable into an Equation

Example
Three times a number that is two less than the number A is the same as two times a number that is one more than A.
What is the number A?

Solution Strategy
The task here is twofold.
1) *Translate* this sentence into an algebraic equation in the variable A.
2) *Solve* the equation for the variable A.

Translation:
Begin by focusing on the phrase

"is the same as" .

This phrase corresponds to the "=" sign, and divides the sentence in half. The information before the phrase sits on one side of the equation, and the information after sits on the other:

$$\left[\begin{array}{c}\text{three times a number that is}\\ \text{two less than the number A}\end{array}\right] = \left[\begin{array}{c}\text{two times a number that is}\\ \text{one more than A}\end{array}\right].$$

Now consider the first phrase

"three times a number that is two less than the number A"

In this phrase *"three times"* means multiplication by 3, and *"a number that is two less than the number A"* means $(A-2)$. Therefore,

$$\left[\begin{array}{c}\text{three times a number that is}\\ \text{two less than the number A}\end{array}\right] = 3(A-2)\ .$$

Next, consider the phrase

"two times a number that is one more than A".

As above, the phrase *"two times"* means multiplication by 2, and *"a number that is one more than the number A"* means $(A+1)$. Therefore,

$$\left[\begin{array}{c}\text{two times a number that is}\\ \text{one more than A}\end{array}\right] = 2(A+1)\ .$$

Putting this all together we get

$$3(A-2) = \left[\begin{array}{c}\text{three times a number that is}\\ \text{two less than the number A}\end{array}\right] = \left[\begin{array}{c}\text{two times a number that is}\\ \text{one more than A}\end{array}\right] = 2(A+1)\ ,$$

or simply,

$$3(A-2) = 2(A+1)\ !$$

This is the equation we have been looking for. It translates the sentence into an algebraic equation in the variable A.

Solving the equation $3(A-2) = 2(A+1)$.

3(A-2)	=	2(A+1)	
3A-6	=	2A+2	(expand both sides of the equation)
+6		+6	(move all numbers to the right by adding 6 to both sides)
3A	=	2A+8	
-2A		-2A	(move all variables to the left by subtracting 2A from both sides)
A	=	8	

Now check the solution by plugging $A = 8$ back into the equation $3(A-2) = 2(A+1)$.

$$3(8-2) = 3\times 6 = 18 \quad \text{and} \quad 2(8+1) = 2\times 9 = 18\ .$$

This all checks out so we have successfully solved this problem.

Now try your hand at the following problems. They are not all exactly the same as the example problem, but they are similar. The problems get more difficult as you proceed.

1. Two times the number A minus four is equal to fourteen.

   What is A?

2. Three less than three times the number Z is four less than ten.

   What is Z?

3. One hundred minus five times the number N equals one fourth of one hundred.

   What is N?

4. The number K minus ten is one half of itself.

   What is K?

5. Three times the number that is three less than the number T is twenty one.

   What is T?

6. Four times the number that is 4 less than the number P is the same as the number that is twice the number that is two more than P.

   What is P?

7. A third of the number that is 20 more than the number R is one fifth the number that is fifty more than R.

   What is R?

8. One hundred minus twice a number that is 10 less than U is the same as the number that is U less than 20.

   What is U?

9. Five times the number S minus twice the number that is three more than S equals three times the number that is twice S less than one.

   What is S?

Solutions to the odd numbered problems.
1. $A = 9$, 3. $N = 15$, 5. $T = 10$, 7. $R = 25$, 9. $S = 1$