

Contents

Ordinary Differential Equations	1
Existence and Uniqueness Theory	1
Reduction to First-Order Systems	1
Initial-Value Problems for First-order Systems	2
The Contraction Mapping Fixed-Point Theorem	4
Local Existence and Uniqueness for Lipschitz f	5
The Picard Iteration	7
Local Existence for Continuous f	9
The Cauchy-Peano Existence Theorem	10
Uniqueness	12
Uniqueness for Locally Lipschitz f	14
Comparison Theorem for Nonlinear Real Scalar Equations	15
Continuation of Solutions	17
Continuation at a Point	17
Global Continuation	18
Continuation for Autonomous Systems	19
Application of continuation theorem to linear systems	20
Continuity and Differentiability of Solutions	21
Fundamental Estimate	21
Continuity with Respect to Parameters and Initial Conditions	23
Transforming “initial conditions” into parameters	23
Transforming Parameters into “Initial Conditions”	24
The Equation of Variation	25
Differentiability	26
Nonlinear Solution Operator	28
Group Property of the Nonlinear Solution Operator	29
Special Case — Autonomous Systems	29
Linear ODE	30
Adjoint Systems	34
Normalized Fundamental Matrices	34
Inhomogeneous Linear Systems	35
Constant Coefficient Systems	36
Application to Nonlinear Solution Operator	38
Rate of Change of Volume in a Flow	39

Linear Systems with Periodic Coefficients	40
Linear Scalar n^{th} -order ODEs	40
Linear Inhomogeneous n^{th} -order scalar equations	43
Linear n^{th} -order scalar equations with constant coefficients	44
Lyapunov Stability	46
Lyapunov Functions	49
Introduction to the Numerical Solution of IVP for ODE	53
Grid Functions	53
Explicit One-Step Methods	53
Local Truncation Error	56
Convergence Theorem for One-Step Methods	57
Explicit Runge-Kutta methods	59
Attainable Orders of Accuracy for Explicit RK methods	61
Linear Difference Equations (constant coefficients)	62
Linear Multistep Methods (LMM)	64
Lebesgue Integration on \mathbb{R}^n	73
Properties of Lebesgue measure	76
Sets of Measure Zero	76
Characterization of Lebesgue measurable sets	77
Invariance of Lebesgue measure	78
Measurable Functions	78
Integration	79
General Measurable Functions	80
Properties of the Lebesgue Integral	81
Comparison of Riemann and Lebesgue integrals	81
Convergence Theorems	82
“Multiple Integration” via Iterated Integrals	84
L^p spaces	85
Completeness	87
Locally L^p Functions	87
Continuous Functions not closed in L^p	87
L^p convergence and pointwise a.e. convergence	88
Change of variables	89
Intuition for growth of functions in $L^p(\mathbb{R}^n)$	90
Polar Coordinates in \mathbb{R}^n	90
Hilbert Spaces	93
Orthogonal Projections onto Closed Subspaces	96
Bounded Linear Functionals and Riesz Representation Theorem	97
Adjoint Operator	98
Strong convergence/Weak convergence	98
Orthogonal Sets	99
Orthonormal Sets	99
Norm Convergence of Fourier Series	102

Cardinality of Orthonormal Bases	102
Fourier Series	105
Fourier Coefficients	106
Absolutely Convergent Fourier Series	109
Decay of Fourier Coefficients \leftrightarrow Smoothness of f	109
Vibrating Strings and Heat Flow	111
Solutions of $u_{tt} = u_{xx}$	112
Initial-Boundary Value Problem (IBVP)	114
Heat Flow	116