

## Problem Set 3: LP Preprocessing

(1) Use preprocessing to simplify the following LPs.

(a)

$$\begin{aligned}
 &\text{maximize} && 2x_1 + x_2 - x_3 \\
 &\text{subject to} && 5x_1 - 2x_2 + 8x_3 \leq 15 \\
 &&& 8x_1 + 3x_2 - x_3 \geq 9 \\
 &&& x_1 + x_2 + x_3 \leq 6 \\
 &&& 0 \leq x_1 \leq 1 \\
 &&& 0 \leq x_2 \leq 1 \\
 &&& 1 \leq x_3
 \end{aligned}$$

(b) Refine (a) under the additional condition that  $x \in \mathbb{Z}^3$ .

(c)

$$\begin{aligned}
 &\text{maximize} && 2x_1 + x_2 - x_3 \\
 &\text{subject to} && 5x_1 - 2x_2 + 8x_3 \leq 15 \\
 &&& 8x_1 - 3x_2 + x_3 \geq 9 \\
 &&& x_1 - x_2 + x_3 \leq 6 \\
 &&& -0.2 \leq x_1 \leq 1 \\
 &&& 0 \leq x_2 \leq 1.5 \\
 &&& 0.5 \leq x_3 \leq 2.5
 \end{aligned}$$

(d) Refine (c) under the additional condition that  $x \in \mathbb{Z}^3$ .

(2) Determine additional *logical inequalities* to simplify the following 0-1 programming problems (BIPs).

(a)

$$\begin{aligned}
 &\text{maximize} && 5x_1 - 7x_2 - 10x_3 + 3x_4 - 5x_5 \\
 &\text{subject to} && x_1 + 3x_2 - 5x_3 + x_4 + 4x_5 \leq 0 \\
 &&& -2x_1 - 6x_2 + 3x_3 - 2x_4 - 2x_5 \leq -4 \\
 &&& 2x_2 - 2x_3 - x_4 + x_5 \leq -2 \\
 &&& x \in \{0, 1\}^5 .
 \end{aligned}$$

(b)

$$\begin{aligned}
 &\text{maximize} && x_1 + 2x_2 - 2x_3 + x_4 \\
 &\text{subject to} && 7x_1 + 3x_2 - 4x_3 - 2x_4 \leq 1 \\
 &&& -2x_1 + 7x_2 + 3x_3 + x_4 \leq 6 \\
 &&& -2x_2 - 3x_3 - 6x_4 \leq -5 \\
 &&& 3x_1 + x_2 - 3x_3 + x_4 \geq -1 \\
 &&& x \in \{0, 1\}^4 .
 \end{aligned}$$