Problem Set 2: The Branch and Bound Algorithm

(1) Solve the following integer knapsack problems by the branch and bound algorithm.

(a)

$$\begin{array}{ll} \text{maximize} & 8x_1 + 11x_2 + 6x_3 \\ \text{subject to} & 5x_1 + 7x_2 + 4x_3 \leq 14 \\ & x \in \{0,1\}^3 \end{array}$$

(b)

$$\begin{array}{ll} \text{maximize} & 10x_1 + 12x_2 + 7x_3 \\ \text{subject to} & 4x_1 + 5x_2 + 3x_3 \leq 10 \\ & x_1, x_2 \in \mathbb{Z}_+,\ x_3 \in \{0,1\} \end{array}$$

(2) Solve the following integer LPs both graphically and by the branch and bound algorithm.

(a)

$$\begin{array}{ll} \text{maximize} & x_1 + x_2 \\ \text{subject to} & 2x_1 + 5x_2 \leq 16 \\ & 6x_1 + 5x_2 \leq 30 \\ & x \in \mathbb{Z}_+^2 \end{array}$$

(b) Repeat problem (a) with $x_1 \in \mathbb{Z}_+$ and $x_2 \in \mathbb{R}_+$.

(c)

$$\begin{array}{lll} \text{maximize} & 9x_1 + 5x_2 & \\ \text{subject to} & 4x_1 + 9x_2 & \leq 35 \\ & 3x_1 + 2x_2 & \leq 19 \\ & x_1 - 3x_2 & \geq 1 \\ & x_1 & \leq 6 \\ & x \in \mathbb{Z}_+^2 & \end{array}$$

(3) Consider an integer linear program of the form

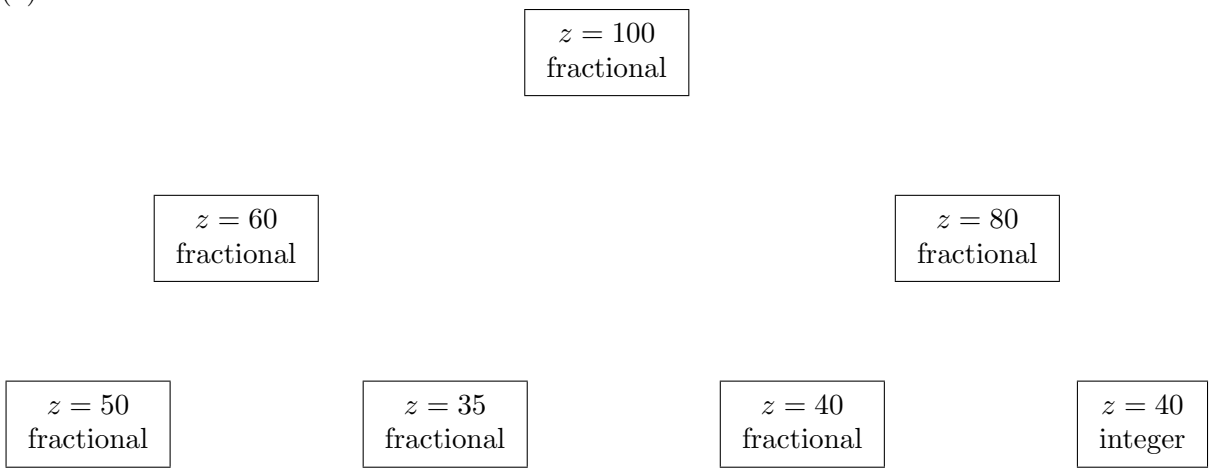
$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & Ax \leq b,\ 0 \leq x,\ x \in \mathbb{Z}^n \end{array}$$

where $A \in \mathbb{Z}^{m \times n}$, $b \in \mathbb{Z}^m$, and $c \in \mathbb{R}^n$. Below are two potential initial portions of the branch and bound tree generated for this problem using LP relaxations, where each box gives the optimal value z for the LP relaxation associated with that box and an indication of whether the LP relaxation gave an integer or fractional solution. With this information answer the following questions for each of the trees below.

(a) What leaves can be pruned? (mark a prune by cutting the linking arc)
(b) What are the current best upper and lower bounds?
(c) How should the branch and bound algorithm proceed from here?

Note:*Please put the lines in between the boxes to complete the tree structure. Sorry, I'm still working on a tool that will do this in Latex.*

(a)



(b)