The Two Phase Simplex Algorithm

Solve the following LPs using the two phase simplex algorithm.

1.

$$
\begin{aligned}
\text{maximize} \quad & 3x_1 + x_2 \\
\text{subject to} \quad & x_1 - x_2 \leq -1 \\
& -x_1 - x_2 \leq -3 \\
& 2x_1 + x_2 \leq 4 \\
& 0 \leq x_1, \ x_2
\end{aligned}
$$

Solution: $(1, 2)$

2.

$$
\begin{aligned}
\text{maximize} \quad & 3x_2 + x_3 \\
\text{subject to} \quad & x_1 + 2x_2 + x_3 \leq 2 \\
& 2x_1 + x_2 - x_3 \leq -1 \\
& 3x_1 + 2x_2 + x_3 \leq 3 \\
& 0 \leq x_1, \ x_2, \ x_3
\end{aligned}
$$

Solution: $(0, 1/3, 4/3)$

3.

$$
\begin{aligned}
\text{maximize} \quad & -2x_1 - 2x_2 - x_3 \\
\text{subject to} \quad & x_1 - 2x_2 + x_3 \leq -2 \\
& 2x_1 - 2x_3 \leq 0 \\
& -x_1 + x_2 \leq -1 \\
& 0 \leq x_1, \ x_2, \ x_3.
\end{aligned}
$$

4.

$$
\begin{aligned}
\text{maximize} \quad & x_1 + x_2 + x_3 \\
\text{subject to} \quad & -2x_1 + 3x_2 + x_3 \leq -1 \\
& x_1 + 3x_2 - x_3 \leq -1 \\
& 2x_1 - x_2 - 3x_3 \leq -6 \\
& 0 \leq x_1, \ x_2, \ x_3.
\end{aligned}
$$

Solution: This problem is feasible, but unbounded.

5.

$$
\begin{aligned}
\text{maximize} \quad & -x_1 + x_2 + x_3 + 4x_4 \\
\text{subject to} \quad & 2x_1 + x_2 + x_4 \leq 3 \\
& -2x_1 - x_2 + x_3 + 4x_4 \leq -2 \\
& 2x_1 - 2x_2 + x_3 + x_4 \leq -4 \\
& 0 \leq x_1, \ x_2, \ x_3, \ x_4.
\end{aligned}
$$

Solution: $(0, 3, 1, 0)$

6.

$$\text{maximize} \quad 2x_1 \qquad\qquad - \quad x_3 \quad + \quad x_4$$

$$\begin{array}{rl}
\text{subject to} & \begin{aligned}
x_1 + x_2 \qquad\qquad\quad - x_4 &\le 7 \\
x_1 - x_2 + x_3 + x_4 &\le -2 \\
0 \le x_1,\ x_2,\ x_3,\ x_4. &
\end{aligned}
\end{array}$$

Solution: $(0, 5, 0, 3)$.

7.

$$\text{maximize} \quad x_1 \ + \ x-2 \ + \ x_3 \ + \ x_4$$

$$\begin{array}{rl}
\text{subject to} & \begin{aligned}
-x_1 + 2x_2 + x_3 + x_4 &\le 3 \\
x_1 - 2x_2 - x_3 + 2x_4 &\le -1 \\
2x_1 + x_2 - x_3 \qquad\quad &\le -2 \\
0 \le x_1,\ x_2,\ x_3,\ x_4. &
\end{aligned}
\end{array}$$

Solution: $(1, 0, 4, 0)$.