Simplex Algorithm for Problems in Standard Form and having Feasible Origin

Solve the following LPs using the simplex algorithm. All of the problems below are in standard form and have feasible origin.

1.
$$\begin{array}{lrcrcrcl}
\text{maximize} & 4x & + & 3y & + & 2z & & \\
\text{subject to} & x & & & + & z & \leq & 2 \\
& -x & - & y & + & z & \leq & 1 \\
& x & + & y & + & z & \leq & 3 \\
& 0 & \leq & x, & y, & z & &
\end{array}$$

Solution: $(2,1,0)$, optimal value $= 11$.

2.
$$\begin{array}{lrcrcrcl}
\text{maximize} & 4x & + & 2y & + & 2z & & \\
\text{subject to} & x & + & 3y & - & 2z & \leq & 3 \\
& 4x & + & 2y & & & \leq & 4 \\
& x & + & y & + & z & \leq & 2 \\
& 0 & \leq & x, & y, & z & &
\end{array}$$

Solution: $(1,0,1)$, optimal value $= 6$.

3.
$$\begin{array}{lrcrcrcr}
\text{maximize} & -7x_1 & + & 9x_2 & + & 3x_3 & & \\
\text{subject to} & 5x_1 & - & 4x_2 & - & x_3 & \leq & 10 \\
& x_1 & - & x_2 & & & \leq & 4 \\
& -3x_1 & + & 4x_2 & + & x_3 & \leq & 1 \\
& 0 & \leq & x_1, & x_2, & x_3. & &
\end{array}$$

Solution: $(4,0,13)$, optimal value $= 11$.

4.
$$\begin{array}{lrcrcrcrcrcl}
\text{maximize} & 7x_1 & + & 6x_2 & + & 5x_3 & - & 2x_4 & + & 3x_5 & & \\
\text{subject to} & x_1 & + & 3x_2 & + & 5x_3 & - & 2x_4 & + & 2x_5 & \leq & 4 \\
& 4x_1 & + & 2x_2 & - & 2x_3 & + & x_4 & + & x_5 & \leq & 3 \\
& 2x_1 & + & 4x_2 & + & 4x_3 & - & 2x_4 & + & 5x_5 & \leq & 5 \\
& 3x_1 & + & x_2 & + & 2x_3 & - & x_4 & + & 2x_5 & \leq & 1 \\
& 0 & \leq & x_1, & & x_2, & & x_3, & & x_4, & & x_5.
\end{array}$$

Solution: $(0,4/3,2/3,5/3,0)$, optimal value $= 8$.