The Dual Simplex Algorithm

Solve the following LPs using the dual simplex algorithm.

1.

$$\begin{array}{lrcrcrcr} \text{minimize} & x_1 & + & 4x_2 & + & 3x_3 & & \\ \\ \text{subject to} & x_1 & - & 2x_2 & - & x_3 & \leq & 1 \\ & -x_1 & - & x_2 & - & 2x_3 & \leq & -3 \\ & x_1 & - & x_2 & + & x_3 & \leq & 2 \\ & 0 & \leq & x_1, & x_2, & x_3 & & \end{array}$$

Solution: $(x_1, x_2, x_3) = (1, 0, 1)$

2.

$$\begin{array}{lrcrcrcr} \text{maximize} & -x_1 & - & 2x_2 & - & 3x_3 & & \\ \\ \text{subject to} & 2x_1 & + & x_2 & - & x_3 & \leq & -1 \\ & -x_1 & + & 2x_2 & & & \leq & -1 \\ & x_1 & - & 2x_2 & + & x_3 & \leq & 5 \\ & 0 & \leq & x_1, & x_2, & x_3. & & \end{array}$$

Solution: $(x_1, x_2, x_3) = (1, 0, 3)$

3.

$$\begin{array}{lrcrcrcr} \text{minimize} & 2x_1 & + & 2x_2 & + & x_3 & & \\ \\ \text{subject to} & x_1 & - & 2x_2 & + & x_3 & \leq & -2 \\ & 2x_1 & & & - & 2x_3 & \leq & 0 \\ & -x_1 & + & x_2 & & & \leq & -1 \\ & 0 & \leq & x_1, & x_2, & x_3. & & \end{array}$$

Solution: The primal is infeasible.

4.

$$\begin{array}{lrcrcrcrcr} \text{minimize} & x_1 & + & x_2 & + & x_3 & + & x_4 & & \\ \\ \text{subject to} & 3x_1 & - & x_2 & & & - & 2x_4 & \leq & -2 \\ & -2x_1 & + & 2x_2 & + & 4x_3 & & & \leq & -2 \\ & & & x_2 & - & 2x_3 & - & x_4 & \leq & -3 \\ & 0 & \leq & x_1, & x_2, & x_3, & & x_4. & & \end{array}$$

Solution: $(x_1, x_2, x_3, x_4) = (1, 0, 0, 3)$