Midterm Description — Math 408 — Winter 2004

The midterm will cover 4 topics:

1. Present value and internal rate of return.
2. Fixed income securities, Bonds in particular.
3. Linear programming
4. Unconstrained Optimization and Convexity

The midterm will have 4 questions. One question each for each of the 4 areas listed above in that order. The questions may be simple vocabulary word like questions, such as

State and prove the Weak Duality Theorem of Linear Programming.

They may be computational in nature, such as solving a two dimentional LP graphically. They may require the modeling of a problem such present value or LP modeling.

Attached to this document you will find a sample midterm exam to give you some idea of scope and type of question that you will be asked.

One page of hand written notes will be allowed during the exam. You may write on both sides of the page.

MATH 408 SAMPLE MIDTERM **NAME (Please print):** ______________________

February 11, 2004

There are 4 problems. Stop now and make sure you have all problems. If you do not have them all, request a new copy of the midterm. Each problem on the exam is worth 50 points for a total of 200 points. Show all of your work and follow the directions provided. Partial credit will be given for partial solutions.

CALCULATORS ARE NOT ALLOWED!

One page of hand written notes are allowed. Both side of the page can be written on.

Problem	Score
1	
2	
3	
4	
Total	

1. (a) Let $x \in I\!R$. Show that

$$\sum_{k=0}^{n} x^k = \frac{1 - x^{(n+1)}}{1 - x} \ .$$

(b) A lottery advertises that it pays the winner $\$1,000,000$. However, this prize money is paid in 20 anual installments of $\$50,000$ each with the first installment paid on the date that the winning ticket is cashed in. At a prevailing interest rate of 5%, write a numerical expression for the present value of the lottery prize on the date that it is cashed in. You must use Part (a) above to reduce the complexity of this numerical expression.

2. XXX Corporation wishes to issue a callable bond at par where current yields are 5% (that is, XXX Corp. wishes to issue a 5% callable bond). To call the bond, XXX Corp. must pay a premium. The terms of the bond require that the premium needs to be declared at the outset. XXX Corp. determines that it should have the option to call the bond after year 3 if the yield on this type of bond rises by more that 2% in that time. Derive a formula for the premium that justifies this calling strategy where the premium is to be determined as a multiple of the face value of the bond.

3. A municipality has the following schedule of liabilities:

Year	2004	2005	2006	2007	2008	2009	2010	2011
Dollars	12,000	18,000	20,000	20,000	16,000	15,000	12,000	10,000

The bonds available for purchase today are givenin the following table. All bonds have a face value of \$100. The coupon figure is annual. For example, bond 5 costs \$98 today and pays back \$4 in 2004, \$4 in 2005, \$4 in 2006, and \$104 in 2007. All of these bonds are widely available and can be purchased in large quantities at the stated prices.

Bond	1	2	3	4	5	6	7	8	9	10
Price	102	99	101	98	98	104	100	101	102	104
Coupon	5%	4%	5%	4%	4%	5%	3%	4%	5%	5%
Maturity	2004	2005	2005	2006	2007	2008	2008	2009	2010	2011

Formulate a linear program to find the least cost portfolio of bonds to purchase today to meet the obligations of the municipality over the next 8 years. Assume that any surplus cash in a given year can be re-invested for a year at annual rate of 6%.

4. (a) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function and let $\phi : \mathbb{R} \to \mathbb{R}$ be a non-decreasing convex function. Show that the function $g : \mathbb{R}^n \to \mathbb{R}$ defined by $g(x) = \phi(f(x))$ is also a convex function.

(b) Show that the function $f(x) = \mathrm{e}^{\|x\|}$ is a convex function.