

There are 2 problems. Stop now and make sure you have both problems. If you do not have them both, then request a new quiz. The first problem is worth 30 points and the second is worth 40 points for a total of 70 points. Show all of your work and follow the directions provided. Partial credit will be given for partial solutions. *CALCULATORS ARE NOT ALLOWED!*

| Problem | Score |
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1. [a](15 points) Let A be a linear transformation from \mathbb{R}^n to \mathbb{R}^m with $m < n$. Give the definition for the *null space* of A and provide a simple lower bound for its dimension.

[b](15 points) Let $A \in \mathbb{R}^{m \times n}$ show that $\text{Null}(A) = \text{Null}(A^T A)$.

2.

[a] Consider the system

$$\begin{aligned} -x_1 & \quad \quad + 4x_3 = 200 \\ -x_1 + x_2 + 9x_3 & = 200 \\ 2x_1 - x_2 + 7x_3 & = 200 . \end{aligned}$$

(i)(15 points) Write the augmented matrix corresponding to this system.

(ii)(15 points) Reduce the augmented system in part (a) to echelon form.

(iii)(10 points) Describe the set of solutions to the given system.