MATH 408 WINTER 2010
SECOND PROGRAMMING ASSIGNMENT

Due Wednesday March 10

GENERAL GUIDELINES:

1. All programs must be written in MATLAB. You may use any machine at your disposal that runs MATLAB. Accounts on the math sciences computing center machines are available.

2. The program should be well documented and all output should be clearly labeled for ease of reference.

3. The interpretation of the numerical output will count for 25% of the grade on the computing projects. That is, you must observe and explain what happened in your numerical experiments in light of the theory we have studied.

4. Turn in the write-up of your interpretations, your program output, and your program listings (in that order), attached together, by the beginning of class on the due date. Please separate the pages of the computer output and trim your output down to 8\text{1\over 2} by 11. (Plan for this as you format your output.)

5. Please start early. No late programming sets will be accepted. Leave time to interpret your results.

INSTRUCTIONS FOR THE SECOND ASSIGNMENT:

1. Write a Matlab implementation of the conjugate gradient algorithm to solve linear least squares problems of the form
   \[ \min \frac{1}{2} \| Ax + b \|_2^2. \]

   The algorithm should be implemented as a iterative procedure that terminates when one of the following three conditions is satisfied:
   
   (a) \( \| Ax + b \| \leq \epsilon \)
   
   (b) \( \| x_{k+1} - x_k \| \leq \epsilon \)
   
   (c) the number of iterations exceeds 2n, where \( A \in \mathbb{R}^{m \times n} \).

   Then use this code to determine the polynomial of degree 6 that best fits the data
   
   \[ y_i = \sin x_i, \text{ with } x_i := -1 + i/5 \text{ for } i = 0, 1, \cdots, 10, \]

   in the least squares sense. Initialize at the two starting points \( p^0 = 0 \) and \( p^0 = \text{ones}(7, 1) \), and use the stopping tolerance \( \epsilon = 10^{-5} \). Explain the difference in performance between the two starting points using your knowledge of what the theory tells you about how it should perform.

   Recall that this polynomial fitting problem requires that you solve a linear least squares problem of the form
   
   \[ \min_{p \in \mathbb{R}^7} \frac{1}{2} \| Xp - y \|_2^2, \]
where $X$ is the Vandermonde matrix associated with the points $x_i$, $y$ is the data vector defined above, and $p = (p_0, p_1, \ldots, p_6)^T$ is the vector of coefficients for the polynomial

$$p_0 + p_1 x + p_2 x^2 + \cdots + p_6 x^6.$$

2. Write a matlab routine to implement both Newton’s method and the inverse Broyden’s method for solving the nonlinear equation $g(x) = 0$ where

$$g(x_1, x_2) = \begin{bmatrix} e^{x_1-1} + x_2^3 - 2 \\ x_1^2 + x_2^2 - 2 \end{bmatrix}.$$

Initialize at the point $x_0 = (1.5, 2)^T$ with $J_0 = g'(x_0)^{-1}$ in the case of Broyden’s method. Use the termination tolerance $\epsilon = 10^{-5}$.

On termination of all routines report the following information.

**termination printout:**

1. A declaration of why the procedure terminated.
2. The value of $\|Ax + b\|$ or $\|g(x)\|$.
3. The total number of iterations.
4. The solution $p$ or $x$.