MATH 408 SPRING 2014
PROGRAMMING PROJECT

Due FRIDAY, JUNE 6

GENERAL GUIDELINES:

1. All programs must be written in MATLAB. You may use any machine at your disposal that runs MATLAB. Accounts on the math sciences computing center machines are available.

2. The program should be well documented and all output should be clearly labeled for ease of reference.

3. The interpretation of the numerical output will count for 25% of the grade on the computing projects. That is, you must observe and explain what happened in your numerical experiments in light of the theory we have studied.

4. Turn in the write-up of your interpretations, your program output, and your program listings (in that order), attached together, by the beginning of class on the due date. Please separate the pages of the computer output and trim your output down to $8\frac{1}{2}$ by 11. (Plan for this as you format your output.)

5. Please start early. No late programming sets will be accepted. Leave time to interpret your results.

INSTRUCTIONS:

Write a Matlab implementation of the conjugate gradient algorithm to solve quadratic optimization problem

$$\min \frac{1}{2}x^T Hx + g^T x \ ,$$

where $H \in \mathbb{R}^{n\times n}$ and $g \in \mathbb{R}^n$. The algorithm should be implemented as a iterative procedure that terminates when one of the following three conditions is satisfied:

1. $\|Hx + g\| \leq 10^{-12}$

2. $\|x_{k+1} - x_k\| \leq 10^{-12}$

3. the number of iterations exceeds $10n$, where $A \in \mathbb{R}^{m\times n}$.

Your code should report back

1. The reason for termination.

2. The last iterate $\bar{x}$.

3. The number of iterations and the values $\|H\bar{x} + g\|$ and $\frac{1}{2}\bar{x}^T H\bar{x} + g^T\bar{x}$.

Apply your code to solve the following two problems. Do NOT forget to discuss what you observe in the light of what the theory tells you. In particular, the theory states that only n iterations are required to obtain the global solution. You need to compare this information with the observes number of iterations required to terminate from each starting point and provide a plausible reason for what they differ the way that they do.

1. Apply the conjugate gradient algorithm initialized at $x^0 = (0,0,0)^T$ and at $x^0 = (-2,-1,1)$ to solve the problem $\min_{x\in\mathbb{R}^3} \frac{1}{2}x^T Hx + g^T x$, where

$$H = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \quad \text{and} \quad g = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}.$$

2. Consider the linear least squares problem

$$\min \frac{1}{2}\|Ax - b\|_2^2$$

as a quadratic optimization problem.

Then use the conjugate gradient code you have developed for the quadratic optimization problem to determine the polynomial of degree 6 that best fits the data

$$y_i = \sin x_i \ , \ \text{ with } x_i := -1 + i/5 \text{ for } i = 0, 1, \cdots, 10,$$

in the least squares sense. Initialize at the two starting points $p^0 = 0$ and $p^0 = ones(7,1)$, and use the stopping tolerance $\epsilon = 10^{-5}$. Explain the difference in performance between the two starting points using your knowledge of what the theory tells you about how it should perform.

Recall that this polynomial fitting problem requires that you solve a linear least squares problem of the form

$$\min_{p\in\mathbb{R}^7} \frac{1}{2}\|Xp - y\|_2^2,$$

where X is the Vandermond matrix associated with the points x_i, y is the data vector defined above, and $p = (p_0, p_1, \ldots, p_6)^T$ is the vector of coefficients for the polynomial

$$p_0 + p_1 x + p_2 x^2 + \cdots + p_6 x^6 \ .$$