SOLUTIONS FOR THE SILICON CHIP PRODUCTION PROBLEM

Initial Tableau

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b
raw wafers	100	100	100	100	1	0	0	0	4000
etching	10	10	20	20	0	1	0	0	600
lamination	20	20	30	20	0	0	1	0	900
testing	20	10	30	30	0	0	0	1	700
	2000	3000	5000	4000	0	0	0	0	0

Optimal Tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	b
0.5	1	0	0	.015	0	0	−.05	25
−5	0	0	0	−.05	1	0	−.5	50
0	0	1	0	−.02	0	.1	0	10
0.5	0	0	1	.015	0	−.1	.05	5
−1500	0	0	0	−5	0	−100	−50	−145,000

1. How much must a type 1 chip be sold for in order to make it efficient to produce them?

   ANSWER: $ 54

   SOLUTION TECHNIQUE: First compute the cost to produce 100 type 1 chips.

cost of 100 type 1 chips =	100	(cost of raw chips)
	$+10 \times 40$	(cost of etching time)
	$+20 \times 60$	(cost of lamination time)
	$+20 \times 10$	(cost of testing time)
	———	
	$1900	(Total cost)

   Thus the cost per chip is $ 19. Since revenue=profit+costs, the the current sale price of type 1 chips is $ 39 per chip. The reduced cost of type 1 chips is $ 15. Hence type 1 chips must be sold for $ 54=39+15 per chip in order to make it efficient to produce them.

2. If it is possible to purchase 1000 more raw silicon wafers a) how many would you be willing to purchase, b) what is the most that you would be willing to pay for them, and c) what would be the new production schedule?

   ANSWERS: a) 500 raw chips

   b) no more than $ 6 per chip

   c) $(x_1,\ x_2,\ x_3,\ x_4)$= (0, 32.5, 0, 12.5)

   SOLUTION TECHNIQUE: The shadow price for raw wafers is 5. Hence we are willing to buy at least some wafers at no more than

$5	(shadow price)
+$1	(market price)
—–	
$6	

   each. We need to determine how many we should buy at this price. To do this we need to determine the range values for the raw silicon wafer resource since these range values precisely determine the range of perturbations to this resource for which the shadow price 5 remains unaltered.

   $$\text{(A)}\left\{\begin{array}{l} \text{Determine the range for the raw silicon wafer resource.} \\ \begin{array}{rcrcclcr} 25 & + & .015\theta & > & 0 & \text{implies } \theta & \geq & -5000/3 \\ 50 & - & .05\theta & > & 0 & \text{implies } \theta & \leq & 1000 \\ 10 & - & .02\theta & > & 0 & \text{implies } \theta & \leq & 500 \\ 5 & + & .015\theta & > & 0 & \text{implies } \theta & \geq & -1000/3\ . \end{array}\end{array}\right.$$

   Therefore we are willing to buy 500 of these raw wafers at no more than $ 6 each. Next we must determine how many of the remaining 500 wafers we wish to purchase and at what price. In order to do this we must pass the perturbation parameter θ above 500 and into the next range of values. We then examine the new shadow price and the new range to make our purchasing decision.

Dual simplex pivot on the third row. The pivot element occurs in the fifth column since this column contains the the only negative element in the third row. This pivot brings the slack variable for raw silicon wafers into the basis. Hence the new shadow price is zero. Furthermore, the new solution is necessarily optimal since the pivot element is the only negative element in the pivot row (check the significance of this statement for yourselves).

Therefore, since the new optimal tableau indicates that any raw silicon wafers over 4500 are surplus, we should not purchase any more than 500 of the available 1000 wafers (of course we could purchase them now for less than or equal to current cost of \$ 1 for use next month, but this is beyond the scope of this question). The new optimal production schedule obtained after purchasing the 500 raw silicon wafers is easily computed from (A) by taking $\theta = 500$.

3. A new product is to be considered for production. This chip requires ten hours each of etching, lamination, and testing time per 100 chip batch. If it can be sold for \$ 33.10 per chip, a) is it efficient to produce, and if so, b) what is the new production schedule?

ANSWERS: a) yes

b) $(x_1,\ x_2,\ x_3,\ x_4) = (0,\ 20,\ 15,\ 0)$ and we produce 100×5 of the new chips.

SOLUTION TECHNIQUE: To compute the break even sale price for this new chip it is necessary to compute its production costs and then add this to its break-even profit level. The production costs for 100 of these chips are

$$\begin{array}{rl} 100 & \text{(cost of the raw wafers)} \\ +10 \times 40 & \text{(cost of etching time)} \\ +10 \times 60 & \text{(cost of lamination time)} \\ +10 \times 10 & \text{(cost of testing time)} \\ \text{---} & \\ 1200 & \text{(total cost)} . \end{array}$$

Thus the production costs are \$ 12 per chip. The break even profit level is obtained by pricing out the resources consumed in the production of one of these chips. This yields

$$5 \times 100 + 0 \times 10 + 100 \times 10 + 50 \times 10 = 2000 \ ,$$

or equivalently, \$ 20 per chip. Hence the break even sale price is \$ 32. This is less than the proposed sale price and so the new chip is efficient to produce. The new column for this new chip in what was previously the optimal tableau is given by

$$\begin{bmatrix} Ma_{\text{new}} \\ --- \\ d^T a_{\text{new}} + c_{\text{new}} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} .015 & 0 & 0 & -.05 \\ -.05 & 1 & 0 & -.5 \\ -.02 & 0 & .1 & 0 \\ .015 & 0 & -.1 & .05 \end{pmatrix} \begin{pmatrix} 100 \\ 10 \\ 10 \\ 10 \end{pmatrix} \\ ---------------------- \\ (-5, 0, -100, -50)(100, 10, 10, 10)^T + 2110 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \\ --- \\ 110 \end{bmatrix}$$

where a_{new} denotes the new column and c_{new} denotes the cost coefficient. The resulting tableau is not dual feasible and so we must pivot on this column. Computing ratios we find that the pivot row is the fourth row. Pivoting yields an optimal tableau with solution values given above.