SOLUTIONS TO THE CONCRETE PRODUCTS CORP. QUESTIONS

Initial Tableau

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
batch mixing	1	2	10	16	1	0	0	800
mold vibrating	1.5	2	4	5	0	1	0	1000
inspection	0.5	0.6	1	2	0	0	1	340
	80	140	300	500	0	0	0	0

Optimal Tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

1. We first compute the current sale price and then add to this the reduced cost for type 3 blocks.

$$\begin{aligned}
\text{current sale price} &= \text{profit} + \text{costs} \\
&= 300 + (5 \cdot 10 + 10 \cdot 4 + 10 \cdot 1 + 100) \\
&= 500 \\
\text{break even sale price} &= \text{reduced cost} + \text{current sale price} \\
&= 280 + 500 \\
&= \$780
\end{aligned}$$

2. To solve this problem, we do a range analysis on the type 2 block profit coefficient. In this problem, we describe how such a range analysis is done. Let us denote the profit level for a pallet of type 2 blocks as $140 + \theta$ where the value $\bar{\theta}$ of θ is to be determined so that if $\theta \geq \bar{\theta}$, then type 2 blocks are produced, but if $\theta < \bar{\theta}$, then type 2 blocks are not produced. With the new profit level given in this way, the exact same pivots producing the optimal tableau above now yield the augmented matrix

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	θ	-280	-400	-50	-20	0	-60000

Note that this is not a simplex tableau due to the presence of the θ in the cost row below the second column. Pivoting on this column to eliminate θ yields the tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	$-280-11\theta$	$-400-19\theta$	$-50-1.5\theta$	$-20+\theta$	0	$-60000-200\theta$

(1)

In order that this tableau remain optimal, we must have

$$\begin{aligned}
-280 - 11\theta &\leq 0 \\
-400 - 19\theta &\leq 0 \\
-50 - 1.5\theta &\leq 0 \\
-20 + \theta &\leq 0\ .
\end{aligned}$$

That is, we must have

$$-\frac{400}{19} \leq \theta \leq 20\ .$$

This range analysis indicates that the current basis changes if we allow the sale price of a pallet of type 2 blocks to drop by more than \$21.05.

Let us now use this information to help us answer the second problem. We know that if we drop the sale price of type 2 blocks by more than \$21.05, then the optimal basis changes. This does not necessarily mean that type 2 blocks drop out of production. It might only mean that we produce fewer of them in the optimal production mix. How do we determine which is the case? For this we return to the perturbed tableau (1). Note that if we drop the sale price of type 2 blocks by more than \$21.05, then the coefficient in the objective row under the x_4 variable becomes positive. This indicates that we need to pivot on this column to obtain the new optimal tableau. If we pivot on this column, then the pivot element is the $(1, 4)$ entry in the simplex tableau, i.e. the number 19 in the first row fourth column since this entry yields the smallest ratio. Pivoting on this entry brings x_4 into the basis and takes x_2 out of the basis. Therefore, type 2 blocks will indeed be removed from the optimal production mix.

The current sale price of type 2 blocks is

$$\begin{aligned} \text{sale price} &= \text{profit} + \text{costs} \\ &= 140 + (5 \cdot 2 + 10 \cdot 2 + 10 \cdot 0.6 + 80) \\ &= \$256 \ . \end{aligned}$$

Hence the minimum sale price at which type 2 blocks can be sold and yet be maintained in the optimal production mix is \$234.95.

3. This is a straight forward range analysis question on the batch mixer resource. Here we need to determine the range of the values θ for which $R(b + \theta e_1) \geq 0$, or equivalently,

$$\begin{aligned} 200 + 1.5\theta &\geq 0 \\ 400 - 2\theta &\geq 0 \\ 20 + 0.1\theta &\geq 0 \ . \end{aligned}$$

This yields the range

$$200 \geq \theta \geq -\frac{400}{3} \ .$$

That is, batch mixing hours can only fluctuate between the values 1000 hours and 666 hours and 40 minutes. If we exceed 1000 hours, then type 1 block comes out of the optimal production mix, and if we drop below 666 hours and 40 minutes, then type 2 block drops out of the optimal production mix. Again, this lower range boundary is very tight relative to the number of hours of batch mixing time are available.

4. We have already done the range analysis on this resource in the previous problem. This analysis indicated that we would like to buy at least 200 hours of batch mixing time at no more that

$$\text{current value} + \text{marginal value} = 5 + 50 = \$55 \text{ per hour.}$$

Since the extra hours are being offered for \$30 an hour we will definitely buy at least 200 hours. Should we buy more that 200 hours? For this we need to understand the effects of a dual simplex pivot on the row that becomes negative if we add slightly more than 200 hours of batch mixing time. This is the second row yielding the $(2, 4)$ element -22 as the pivot element. It is easy to see that pivoting on this element returns us to optimality (why?). The new shadow price for batch mixing time is $50 - 2 \cdot \frac{400}{22} = \frac{150}{11}$. Since $5 + \frac{150}{11} < 30$ we do not want to buy any more batch mixing time.

5. In problem 2 above we already performed the range analysis on the objective coefficient for type 2 blocks. Since the change in sale price exceeds \$20 per pallet, the optimal production mix changes. The easiest way to compute the new production mix is to plug the number 30 in for θ in the tableau (1) and then pivot to

optimality. This yields

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-610	-970	-95	10	0	-66000
.5	1	5	8	14	0	0	400
.5	0	-6	-11	-1	1	0	200
.2	0	-2	-2.8	$-.3$	0	1	100
-5	0	-550	-860	-85	0	0	-68000

Therefore, in the new optimal production schedule we only produce 400 pallets of type 2 blocks.

6. Here with we start as though we were going to perform a range analysis on the mold vibrating resource. This yields the parameterized tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	$200 - \theta$
1	0	-12	-22	-2	2	0	$400 + 2\theta$
0	0	0.4	1.6	0.1	-0.4	1	$20 - 0.4\theta$
0	0	-280	-400	-50	-20	0	$-60000 - 20\theta$

Plugging in $\theta = -300$ yields the tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	500
1	0	-12	-22	-2	2	0	-200
0	0	0.4	1.6	0.1	-0.4	1	140
0	0	-280	-400	-50	-20	0	-54000

We need to apply the dual simplex algorithm to recover optimality. Pivoting to optimality on the $(2, 4)$ element takes x_1 out of the basis and brings x_4 into the basis. In the new optimal production mix we make $100/11 = 9 + \frac{1}{11}$ pallets of type 4 blocks and $500 - \frac{1900}{11} = \frac{3600}{11} = 327 + \frac{3}{11}$ pallets of type 2 blocks.

7. The cost of producing a pallet of this new block type is

$$5 \cdot 4 + 10 \cdot 4 + 10 \cdot 1 + 80 = \$150 \ .$$

Pricing out the marginal value of the resources consumed in the production of a pallet of this new block type, we get

$$50 \cdot 4 + 20 \cdot 4 + 0 \cdot 1 = \$280 \ .$$

Therefore, the price at which it becomes efficient to produce this product is $280 + 150 = \$430$.