There are 2 problems. Stop now and make sure you have both problems. If you do not have them both, then request a new quiz. The first problem is worth 30 points and the second is worth 45 points for a total of 75 points. Show all of your work and follow the directions provided. Partial credit will be given for partial solutions. *CALCULATORS ARE NOT ALLOWED!*

Problem   Score

1

2

Total

1. [a](15 points) Under what conditions is the set $S \subset \mathbb{R}^n$ a subspace of $\mathbb{R}^n$.

Solution S is a subspace of $\mathbb{R}^n$ if

   (i) $0 \in S$

   (ii) $x + y \in S$ for all $x, y \in S$.

   (iii) $\alpha x \in S$ for all $\alpha \in \mathbb{R}$ and $x \in S$.

[b](15 points) Let A be a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$ with $m < n$. Give the definition for the *null space* of A and provide a simple lower bound for its dimension.

Solution

$$\mathrm{Nul}(A) = \{x \in \mathbb{R}^n \mid Ax = 0\} \quad \text{and} \quad \dim(\mathrm{Nul}(A)) = n - \dim(\mathrm{Ran}(A)) \geq n - m$$

2. Consider the system

$$
\begin{aligned}
-x_1 \quad\quad + \quad 4x_3 &= 200 \\
-x_1 + x_2 + 9x_3 &= 200 \\
2x_1 - x_2 + 7x_3 &= 200 \ .
\end{aligned}
$$

(a)(15 points) Write the augmented matrix corresponding to this system.

Solution

$$
\left[\begin{array}{ccc|c}
-1 & 0 & 4 & 200 \\
-1 & 1 & 9 & 200 \\
2 & -1 & 7 & 200
\end{array}\right]
$$

(b)(20 points) Reduce the augmented system in part (a) to echelon form.

Solution

-1	0	4	200	
-1	1	9	200	
2	-1	7	200	
1	0	-4	-200	$-r_1$
0	1	5	0	$r_2 - r_1$
0	-1	15	600	$r_3 + 2r_1$
1	0	-4	-200	
0	1	5	0	
0	0	20	600	$r_2 + r_3$

(c)(10 points) Describe the set of solutions to the given system.

Solution

$$
\begin{aligned}
20x_3 &= 600 & \rightarrow & \quad x_3 = 30 \\
x_2 + 5x_3 &= 0 & \rightarrow & \quad x_2 = -5x_3 = -150 \\
x_1 - 4x_3 &= -200 & \rightarrow & \quad x_1 = -200 + 4x_3 = -80
\end{aligned}
$$

Therefore, the solution set consists of the unique point

$$
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -80 \\ -150 \\ 30 \end{pmatrix} .
$$