Math 407: Linear Optimization

Lecture 14: Sensitivity Analysis
Concrete Products Corp

Math Dept, University of Washington

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Concrete Products Corporation has the capability of producing four types of concrete blocks. Each block must be subjected to four processes: batch mixing, mold vibrating, inspection, and yard drying. The plant manager desires to maximize profits during the next month. During the upcoming 30 days, he has 800 machine hours available on the batch mixer, 1000 hours on the mold vibrator, and 340 man-hours of inspection time. Yard-drying time is unconstrained. Taking into consideration depreciated capital investment and maintenance costs, batch mixing time is worth \$5 per hour, mold vibrating time is worth \$10 per hour, and inspection time is worth \$10 per hour, and the materials costs for the blocks are \$50, \$80, \$100, and \$120 per pallet, respectively. The production director has formulated his problem as a linear program.

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	x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
batch mixing	1	2	10	16	1	0	0	800
mold vibrating	1.5	2	4	5	0	1	0	1000
inspection	0.5	0.6	1	2	0	0	1	340
	80	140	300	500	0	0	0	0

Resource costs

$$\begin{aligned} \text{batch mixing} &= \$5\,hr \\ \text{mold vibrating} &= \$10\,hr \\ \text{inspection} &= \$10\,hr \end{aligned}$$

Materials costs: $(x_1, x_2, x_3, x_4) \sim (\$50,\ \$80,\ \$100,\ \$120)$.

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Sale price = profit + costs

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Sale price = profit + costs

$$x_1 \sim 80 + 5 \times 1 + 10 \times 1.5 + 10 \times 0.5 + 50 = \$155$$

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Sale price = profit + costs

$$\begin{aligned} x_1 &\sim 80 + 5 \times 1 + 10 \times 1.5 + 10 \times 0.5 + 50 = \$155 \\ x_2 &\sim 140 + 5 \times 2 + 10 \times 2 + 10 \times 0.6 + 80 = \$256 \end{aligned}$$

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Resource costs

$$\begin{aligned} \text{batch mixing} &= \$5\,hr \\ \text{mold vibrating} &= \$10\,hr \\ \text{inspection} &= \$10\,hr \end{aligned}$$

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Sale price = profit + costs

$$\begin{aligned} x_1 &\sim 80 + 5 \times 1 + 10 \times 1.5 + 10 \times 0.5 + 50 = \$155 \\ x_2 &\sim 140 + 5 \times 2 + 10 \times 2 + 10 \times 0.6 + 80 = \$256 \\ x_3 &\sim 300 + 5 \times 10 + 10 \times 4 + 10 \times 1 + 100 = \$500 \end{aligned}$$

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Resource costs

$$\begin{aligned} \text{batch mixing} &= \$5\,hr \\ \text{mold vibrating} &= \$10\,hr \\ \text{inspection} &= \$10\,hr \end{aligned}$$

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Sale price = profit + costs

$$\begin{aligned} x_1 &\sim 80 + 5 \times 1 + 10 \times 1.5 + 10 \times 0.5 + 50 = \$155 \\ x_2 &\sim 140 + 5 \times 2 + 10 \times 2 + 10 \times 0.6 + 80 = \$256 \\ x_3 &\sim 300 + 5 \times 10 + 10 \times 4 + 10 \times 1 + 100 = \$500 \\ x_4 &\sim 500 + 5 \times 16 + 10 \times 5 + 10 \times 2 + 120 = \$770 \end{aligned}$$

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Optimal tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

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Optimal tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

What is the break-even sale price for one pallet of types 3 and 4 concrete blocks?

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Optimal tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

What is the break-even sale price for one pallet of types 3 and 4 concrete blocks?

$$\text{Break-even sale price} = \text{reduced cost} + \text{current sale price}$$

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Optimal tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

What is the break-even sale price for one pallet of types 3 and 4 concrete blocks?

$$\text{Break-even sale price} = \text{reduced cost} + \text{current sale price}$$
$$\text{Type 3} = \$280 + \$500 = \$780$$

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Optimal tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

What is the break-even sale price for one pallet of types 3 and 4 concrete blocks?

$$\text{Break-even sale price} = \text{reduced cost} + \text{current sale price}$$

$$\text{Type 3} = \$280 + \$500 = \$780$$

$$\text{Type 4} = \$400 + \$770 = \$1170$$

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Optimal tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

What is the minimum price at which type 2 blocks can be sold and and yet maintain them in the optimal production mix?

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Optimal tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

If the 800 machine hours on the batch mixer is uncertain, for what range of hours of batch mixing time is it efficient for the optimal production mix to consist of type 1 and 2 blocks?

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Optimal tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

A competitor has offered the manager additional batch mixing time at \$30 an hour. Neglecting transportation costs, should the manager accept this offer and if so, how many hours of batch mixing time should he purchase at this price?

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Optimal tableau

x_1	x_2	x_3	x_4	x_5	x_6	x_7	b
0	1	11	19	1.5	-1	0	200
1	0	-12	-22	-2	2	0	400
0	0	0.4	1.6	0.1	-0.4	1	20
0	0	-280	-400	-50	-20	0	-60000

The mold vibrator needs major repairs. Consequently, we will lose 300 hours of mold vibrating time this month. What should be the new production schedule for this month ?