The final exam for this course is set to be given on Wednesday, March 19, at 8:30-10:20 am in the same classroom that the course always meets in.

EXAM OUTLINE

The final exam will consist of 5 questions each worth 60 points. The content of each question is as follows.

**Question 1:** This question specifically refers to the content of the *Linear Programming Theory: Review Notes* provided on the course website. In this question you will be asked to state and/or prove one or more of the following six results contained in these notes:

(i) The weak duality theorem (state and prove).
(ii) The two phase simplex algorithm outcomes (state).
(iii) The fundamental theorem of linear programming (state).
(iv) The strong duality theorem (state and prove).
(v) The complementary slackness theorem (state).
(vi) The Fundamental Theorem of Sensitivity Analysis (state).

I emphasize, you are **required** to learn the statements of these results as given in the *Linear Programming Theory: Review Notes*, NOT the statements as given in the text.

**Question 2:** In this question you will be asked to model one or more of the LP models 1–25 given on the class web page.

**Question 3:** In this question you will be given one or more LPs and asked to solve them. You may solve them using any of the techniques developed in class (graphing, the simplex algorithm, the two phase simplex algorithm, the dual simplex algorithm).

You will need to show all of your work to get full credit. In addition, you may be asked to answer a question about the nature of the solution that you have found or the nature of the dual solution.

**Question 4:** In this question you will be asked to do one or more of the following three things:

(i) Put a given LP into standard form.
(ii) Formulate the dual of a given LP without first bringing it to standard form (theorems of the alternative are possible here).
(iii) Use the complementary slackness theorem to verify the optimality of a putative solution to a given LP.
(iv) Use the geometric duality theorem to verify the optimality of a putative solution to a given LP.

**Question 5:** In this problem you will be given an LP model, its initial tableau, and an associated optimal tableau. You will then be asked to answer certain questions about the problem using the techniques of sensitivity analysis.
SAMPLE QUESTIONS

1. Answer each of the following:
   (i) The weak duality theorem. (state and prove)
   (ii) The two phase simplex algorithm outcomes. (state)
   (iii) The fundamental theorem of linear programming. (state)
   (iv) The strong duality theorem. (state and prove)
   (v) The complementary slackness theorem. (state)
   (vi) The Fundamental Theorem of Sensitivity Analysis. (state)

2. A farmer has to purchase the following quantities of fertilizer from four different shops, subject to the following capacities and prices. How can he fulfill his requirements at minimal cost?

<table>
<thead>
<tr>
<th>Fertilizer Type</th>
<th>Minimum Required (tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>185</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
</tr>
<tr>
<td>5</td>
<td>185</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximum (all types combined)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shop Number</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Price in Money Units per Ton of Fertilizer Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>At Shop</td>
</tr>
<tr>
<td>---------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

3. (a) Solve the following LP stating its solution and optimal value.

maximize $4x_1 + 4x_2 + 5x_3 + 3x_4$
subject to
$x_1 + x_2 + x_3 + x_4 \leq 40$
$x_1 + x_2 + 2x_3 + x_4 \leq 40$
$2x_2 + 2x_2 + 3x_3 + x_4 \leq 60$
$3x_1 + 2x_2 + 2x_3 + 2x_4 \leq 50$
$0 \leq x_1, x_2, x_3, x_4.$

(b) State the dual of this LP and give its solution.

4. (a) Formulate a dual for the LP

minimize $c^T x$
subject to $Ax \leq 0$
$Bx = 0,$
where $c \in \mathbb{R}^n$, $A \in \mathbb{R}^{s \times n}$, and $B \in \mathbb{R}^{t \times n}$.

(b) 

maximize $2x_1 - 3x_2 + 10x_3$
subject to $x_1 + x_2 - x_3 = 12$
$x_1 - x_2 + x_3 \leq 8$
$0 \leq x_2 \leq 10$
5. Donna’s Trees is planning their planting strategy for year 2000 Christmas trees. Donna has allocated a total budget of $16000 for this venture. The other limiting factors in her planning strategy are the number of acres available to plant (10 acres), the number of trucks available for shipping the trees to the markets in Seattle (one truck that can make about 20 trips during the critical time frame), and the number of person hours available for harvesting and marketing the trees during the critical time frame (2400 hours). Her plan is to plant Scotch Pine, Douglas Firs, and White Pine, each of which requires different amounts of the necessary resources per acre. For example, Scotch Pine requires about $4000 per acre to produce and market. This figure includes all planting, harvesting, shipping, and marketing costs. The same figure is $10000 for Douglas Fir and $2000 for White Pine. Her target profit figures for each acre of Scotch Pine, Douglas Fir, and White Pine are $6000, $12000, and $5000, respectively. Thus, per acre, their target sale prices for Scotch Pine, Douglas Fir, and White Pine are $10000, $22000, and $7000, respectively. Donna has decided to model her problem as a linear program. The initial tableau associated with this model is as follows:

```
  SP  DF  WP  S1  S2  S3  S4  b
  |   |   |   |   |   |   |   |   |
acres 1 1 1 1 0 0 0 10
budget 4000 10000 2000 0 1 0 0 16000
trucking 4 8 4 0 0 1 0 20
labor 500 1000 400 0 0 0 1 2400
|   |   |   |   |   |   |   |   |
  |   |   |   |   |   |   |   |
```

where the cost coefficient in the z-row represent profit per acre in dollars. After applying the simplex algorithm Donna obtained the following optimal tableau:

```
  SP  DF  WP  S1  S2  S3  S4  Mb
  |   |   |   |   |   |   |   |
1  3  0  0  .0005  -.25  0  3
0 -100 0 0  -.05  -.75  1  100
0 -1  0  1  0  -.25  0  5
0 -1  1  0  -.0005  .5  0  2
0 -1000 0 0  -.5  -1000  0  -28000
```

Use this information to answer the following questions. Treat each question as an isolated event. Do not carry information between questions.

(a) What is the break-even sale price for an acre of Douglas Fir Christmas trees?

(b) The truck used to bring the trees to Seattle can makes only one trip per day. The truck is driven to the tree nursery, the trees are harvested, loaded onto the truck, and the truck returns to Seattle. Donna has enough employees for two truck crews and so is thinking of renting a truck with identical carrying capacity for a few days. Should Donna rent the truck? If she rents the truck, how many days should she rent it for and what is the most she is willing to pay in daily rental fees?

(c) Donna is flexible in adjusting her budget. She is willing to either borrow more money to invest in the Christmas tree production, or she can take some or all of the $16000 and invest it in another project. For each dollar she invests in the other project, she gets two dollars at the end of three years.

i. Should Donna take out a loan, or should she invest some or all of the $16000 in the other project?
ii. If Donna borrows money, how much should she borrow and what is the maximum three year cumulative finance charges that can be incurred and yet maintain the profitability of the loan?
iii. If Donna invests in the other project, how much of the $16000 should she invest?