EXAM OUTLINE

This exam will consist of 5 questions each worth 50 points for a total of 250 points. The content of each question is as follows.

**Question 1:** In the first question you will be asked to recite the definition of one or more terms from the class notes and Chapters 1–3 of the text. The precise list of terms that you need to know can be found in the weekly overviews for weeks 1–6.

**Question 2:** In this question you will be asked to model a given problem as an LP. The particular model will be chosen from the models 1 through 16 found on the class webpage.

**Question 3:** In this question you will be asked to solve a two dimensional LP graphically, state the dual of this LP, and compute the solution to the dual (i.e. compute the marginal values).

**Question 4:** In this question you will be asked to convert an LP to standard form.

**Question 5:** In this question you will be asked to solve an LP using the two phase simplex algorithm (or, only phase 2 if the LP has feasible origin).

**SAMPLE QUESTIONS**

1. (a) What does it mean to say that an LP is unbounded? Provide and example of an unbounded LP.
   (b) State and prove the Weak Duality Theorem of Linear Programming for LPs in standard form.

2. A company needs to lease warehouse storage space over the next 5 months. Just how much space will be required in each of these months is known. However, since these space requirements are quite different, it may be most economical to lease only the amount needed each month on a month-by-month basis. On the other hand, the additional cost for leasing space for additional months is much less than for the first month, so it may be less expensive to lease the maximum amount needed for the entire 5 months. Another option is the intermediate approach of changing the total amount of space leased (by adding a new lease and/or having an old lease expire) at least once but not every month.

   The space requirement (in thousands of square feet) and the leasing costs (in hundreds of dollars) for the various leasing periods are as follows:

<table>
<thead>
<tr>
<th>Month</th>
<th>Required Space</th>
<th>Leasing Period (months)</th>
<th>Cost ($ per 1,000 square feet leased)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30</td>
<td>1</td>
<td>650</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>2</td>
<td>1000</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>3</td>
<td>1,350</td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td>4</td>
<td>1,600</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>5</td>
<td>1,900</td>
</tr>
</tbody>
</table>

   The objective is to minimize the total leasing cost for meeting the space requirement. Formulate the linear programming model for this problem. You do not have to put the LP into standard form.
3. Consider the following LP:

\[
\text{maximize} \quad 4x - y \\
\text{subject to} \quad -2x + y \leq 4 \\
\quad \quad \quad x + y \leq 7 \\
\quad \quad \quad 2x - y \leq 1 \\
\quad \quad \quad x + y \geq 1 \\
\quad \quad \quad x \leq 2, \; 0 \leq y
\]

(a) Solve this LP graphically.
(b) State the dual to this LP.
(c) Compute the solution to the dual.

4. Put the following LP into standard form:

\[
\text{minimize} \quad 4x_1 - 2x_2 + x_3 \\
\quad -x_1 + 3x_2 - x_3 \geq -1 \\
\quad 5x_2 + 3x_3 = 5 \\
\quad x_1 + x_2 + x_3 \leq 1 \\
\quad -1 \leq x_2, \; -2 \leq x_3 \leq 2
\]

5. Solve the following LP using the simplex algorithm in tableau format. State the solution and the associated optimal value.

\[
\text{maximize} \quad 2y + 3z \\
\text{subject to} \quad -x + y - z \leq 1 \\
\quad \quad \quad x - 2y \leq 0 \\
\quad \quad \quad x + 3z \leq 1 \\
\quad \quad \quad 0 \leq x, \; y, \; z
\]