MIDTERM 1 Review

Let $v^1, \ldots, v^k \in \mathbb{R}^n$ and let $V \in \mathbb{R}^{n \times k}$ be the matrix having columns $v^1, \ldots, v^k$. For each item in the left hand column circle the letter of the entries in the right hand column that are equal to it. Note that a given entry in the left hand column may equal several entries on the right or may not equal any entry on the right. One point for each correct answer and minus one point for each incorrect answer. The total score will be the maximum of the sum of the points and zero, but no greater than 5 points.

Ran (V) a b c d e f

Nul (V) a b c d e f

(a) $\left\{\alpha_1 v^1 + \alpha_2 v^2 + \cdots + \alpha_k v^k \,:\, \alpha_1, \alpha_2, \ldots, \alpha_k \in \mathbb{R}\right\}$

(b) $\left\{(\alpha_1, \alpha_2, \ldots, \alpha_k) \,:\, \alpha_1 v^1 + \alpha_2 v^2 + \cdots + \alpha_k v^k = 0,\ \alpha_1, \alpha_2, \ldots, \alpha_k \in \mathbb{R}\right\}$

(c) $\left\{y \in \mathbb{R}^n \,:\, \exists\, x \in \mathbb{R}^k \text{ such that } y = Vx,\ y_1 v^1 + y_2 v^2 + \cdots + y_k v^k = 0\right\}$

(d) $\left\{x \in \mathbb{R}^k \,:\, Vx = 0\right\}$

(e) $\left\{y \in \mathbb{R}^n \,:\, \exists\, x \in \mathbb{R}^k \text{ such that } y = Vx\right\}$

(f) Span $\left[v^1, \ldots, v^k\right]$

Let $A \in \mathbb{R}^{n\times m}$ and let $B \in \mathbb{R}^{n\times m}$ be a matrix obtained by transforming A into echelon form.

(1) What must be true about B for $\operatorname{Ran}(A) = \mathbb{R}^n$?

(2) What must be true about B for $\operatorname{Nul}(A) = \{0\}$?

Let $A = \begin{bmatrix} 1 & 0 & -1 & 2 & 1 \\ 2 & 1 & -1 & 4 & 3 \\ 3 & 1 & -7 & 5 & 5 \end{bmatrix}$.

(1) Is A one-to-one?
(2) Is A onto?

If the following linear system has a nonempty solution set, write the set of solutions in vector form.

$$\begin{array}{rcrcrcrcrcl} x_1 & + & & - & x_3 & + & 2x_4 & + & x_5 & = & 1 \\ 2x_1 & + & x_2 & - & x_3 & + & 4x_4 & + & 3x_5 & = & 1 \\ 3x_1 & + & x_2 & - & 7x_3 & + & 5x_4 & + & 5x_5 & = & 1 \end{array}$$

Let

$$u^1 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, \quad u^2 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \quad \text{and} \quad u^3 = \begin{pmatrix} z_1 \\ -4 \\ z_2 \end{pmatrix}.$$

(1) Find all values z_1 and z_2 such that u^1, u^2, and u^3 do **not** span $\mathbb{R}^3$.
(2) Find all values z_1 and z_2 such that u^1, u^2, and u^3 are linearly independent.