Steps to solving minimum distance problems for lines and planes

Lines: You are given a point on a line $Q \in \mathbb{R}^3$, a non-zero vector parallel to the line $\mathbf{v}$, and some other point $P \in \mathbb{R}^3$ and asked to find (a) the point on the line closest to P, say R, and (b) the distance from P to the line which we define to be the distance between P and R.
We answer both (a) and (b) using the following steps (after drawing the picture if you can).

Step 1: Compute the vector $\overrightarrow{QP}$.
Step 2: Compute the projection of $\overrightarrow{QP}$ onto $\mathbf{v}$:

$$\text{proj}_{\mathbf{v}}\overrightarrow{QP} := \frac{\overrightarrow{QP} \bullet \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}.$$

Step 3: Compute the position vector for the closest point on the line by

$$\overrightarrow{OR} = \overrightarrow{OQ} + \text{proj}_{\mathbf{v}}\overrightarrow{QP}.$$

The the components of $\overrightarrow{OR}$ give the components of the nearest point R.
Step 4: Compute

$$\text{distance from } P \text{ to the line} = |\overrightarrow{PR}| = \sqrt{|\overrightarrow{QP}|^2 - \left|\text{proj}_{\mathbf{v}}\overrightarrow{QP}\right|^2}.$$

Questions:

(i) Show that $Q = R$ if and only if $\overrightarrow{QP} \bullet \mathbf{v} = 0$.
(ii) Show that $P = R$ if and only if $|\overrightarrow{QP} \times \mathbf{v}| = 0$.

Planes: You are given a normal to a plane $\mathbf{n} = < a, b, c >$, a point $Q(x_0, y_0, z_0) \in \mathbb{R}^3$ in the plane, and another point $P(\bar{x}, \bar{y}, \bar{z}) \in \mathbb{R}^3$, and asked (a) to find the point R in the plane that is closer to P than any other point in the plane and (b) to find the distance from P to R which we call the distance from P to the plane.
We answer both (a) and (b) using the following steps (after drawing the picture if you can).

Step 1: Compute the vector $\overrightarrow{QP}$.
Step 2: Compute the projection of $\overrightarrow{QP}$ onto $\mathbf{n}$:

$$\text{proj}_{\mathbf{n}}\overrightarrow{QP} := \frac{\overrightarrow{QP} \bullet \mathbf{n}}{|\mathbf{n}|^2}\mathbf{n}.$$

Step 3: Compute

$$\begin{aligned}\text{distance from } P \text{ to the plane} &= \left|\text{proj}_{\mathbf{n}}\overrightarrow{QP}\right| = |\text{comp}_{\mathbf{n}}\overrightarrow{QP}| \\ &= \frac{|\overrightarrow{QP} \bullet \mathbf{n}|}{|\mathbf{n}|} = \frac{|a\bar{x} + b\bar{y} + c\bar{z} - d|}{\sqrt{a^2 + b^2 + c^2}},\end{aligned}$$

where $d := ax_0 + by_0 + cz_0$ so that the equation for the plane is $ax + by + cz = d$.
Step 4: Compute the position vector for R:

$$\overrightarrow{OR} = \overrightarrow{OP} - \text{proj}_{\mathbf{n}}\overrightarrow{QP}.$$

The components of $\overrightarrow{OR}$ are the components of the nearest point R.

Questions:

(i) Show that $Q = R$ if and only if $\text{proj}_{\mathbf{n}}\overrightarrow{QP} = \overrightarrow{QP}$.
(ii) Show that $P = R$ if and only if $\text{proj}_{\mathbf{n}}\overrightarrow{QP}$ is the zero vector.