



# De Finetti's ultimate failure

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# Does philosophy matter?

“Global temperatures will rise by 1 degree in 20 years **with probability 80%.**”

# Reading suggestions

- “Probability is Symmetry” (KB)  
<http://www.math.washington.edu/~burdzy/Philosophy/book.pdf>
- D.A. Gillies “Philosophical Theories of Probability” Routledge, London, 2000

Classical statistics ~~↔~~ Frequency philosophy

Bayesian statistics ~~↔~~ Subjective philosophy

# Four mature philosophies

Created in twentieth century, randomly ordered

<b>Name</b>	<b>Principal philosopher</b>	<b>What is the nature of probability?</b>
Logical	Rudolf Carnap	Weak implication
Propensity	Karl Popper	Physical property
Frequency	Richard von Mises	Attribute of a sequence
Subjective	Bruno de Finetti	Personal opinion

# Twin philosophies

The frequency and subjective philosophies are the **only** two philosophies which claim that **events do not have probabilities.**



# “Probability does not exist”

(de Finetti)

The subjective philosophy is a religion – its dogmas have to be interpreted.

# Decisions and probability

De Finetti's most scientific and least controversial claim was that the rules of Bayesian inference can be derived from a system of axioms for rational decision making that did not presuppose existence of probability.



# Lindley's challenge

The talk is in part an answer to the following challenge posed by D. Lindley in his "Math review" of M.H. DeGroot's "Optimal Statistical Decisions":

"Many statisticians and decision-theorists will be out of sympathy with the book because it is openly Bayesian. ...

But they would do well to consider the argument dispassionately and consider whether the axioms are acceptable to them. If they are, then the course is clear; if not, then they should say why and then develop their own and the deductions from them."



# From decisions to probability

De Finetti's idea has (at least) two representations.

- The Dutch book argument (popular with philosophers).
- The von Neumann-Morgenstern-Savage system of axioms (formal mathematics).

# Axioms

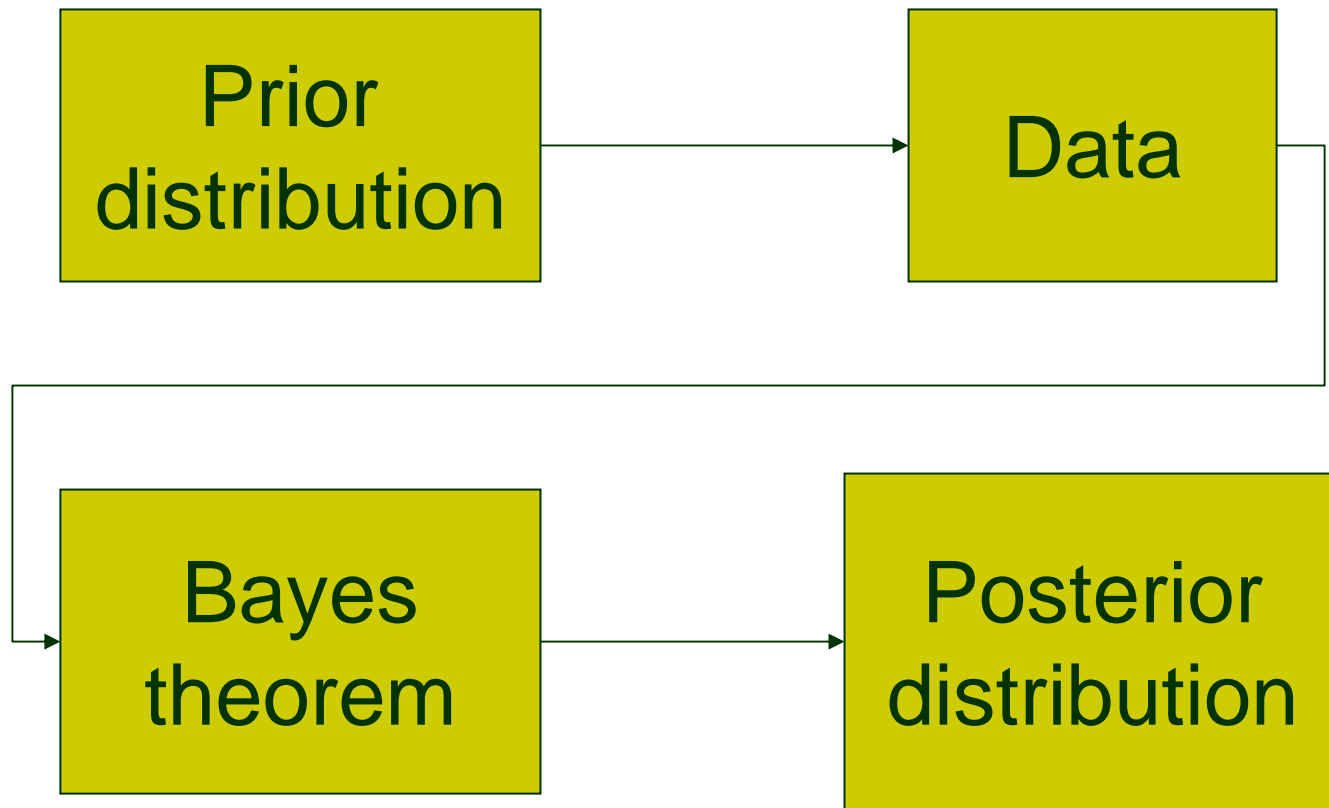
“Decision A is strictly preferable to decision B  
and  
decision B is strictly preferable to decision A”  
is irrational.

- Highest level of abstraction
- Elimination of irrational decision strategies
- No indication of how to order “rational” strategies
- Real axioms are more complicated
- Real people do not follow the axioms

# Existence theorem

**Theorem 1** (de Finetti- von Neumann- Morgenstern-Savage). If a decision strategy is rational then there exist a probability  $P$  and a utility function  $U$  such that decision  $A$  is preferable to decision  $B$  if and only if  $E U(A) > E U(B)$ .

# Bayesian statistics

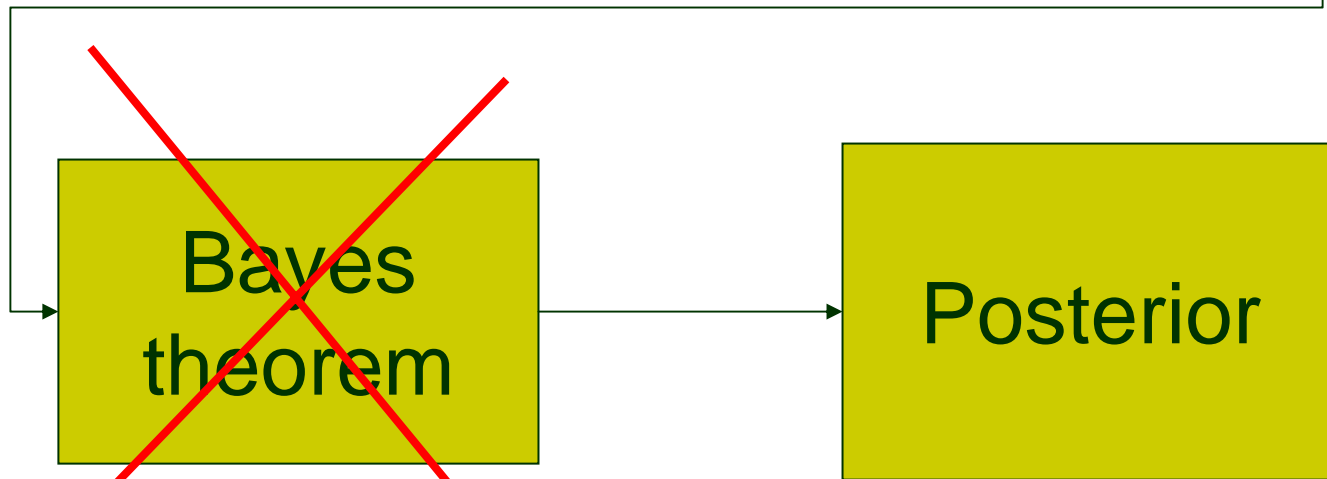


# De Finetti's ultimate failure

Theorem 1 can be used to show that



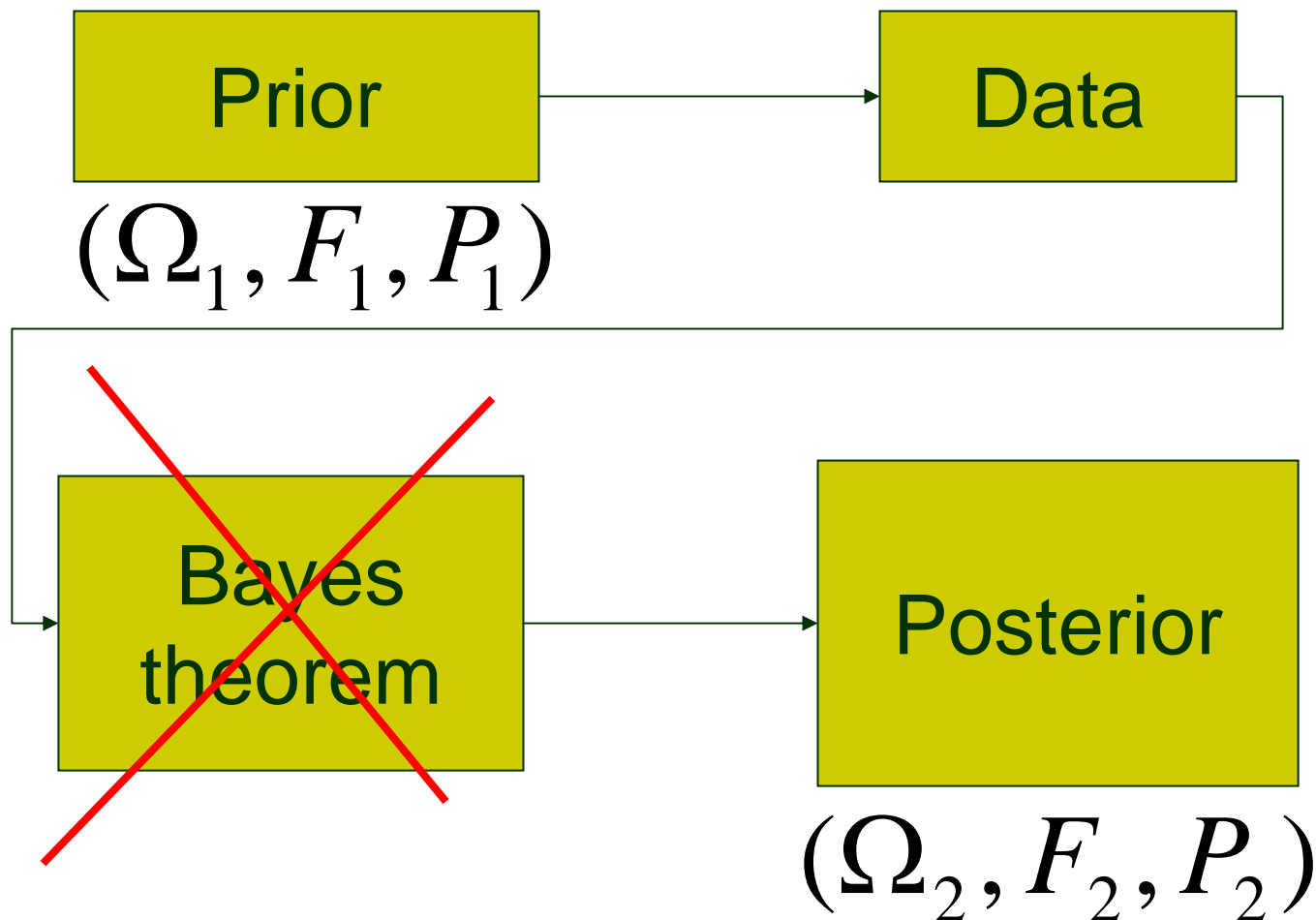
Prior must be a probability distribution.



Posterior must be a probability distribution.

# De Finetti's ultimate failure

Theorem 1 can be used to show that





# Investment strategy (an example)

- Monday: Buy stocks or bonds
- Tuesday: Read newspaper (new data)
- Wednesday: Buy stocks or bonds





The axioms can be applied to actions taken  
on ~~Wednesday~~ ~~Wednesday~~:

“Stocks are strictly better than bonds  
and  
bonds are strictly better than stocks”  
is irrational.

None of the following investment strategies is irrational.

Day	Monday	Wednesday
Preferred investment	Stocks	Stocks
	Stocks	Bonds
	Bonds	Stocks
	Bonds	Bonds



# Separation of decision strategies

The axioms do not specify any relationship between actions taken on Monday and actions taken on Wednesday. Theorem 1 splits in the Bayesian context into two separate theorems – one on the prior actions and the other on the posterior actions.

# Irreparable error

**Theorem 2 (KB).** Suppose strategy  $S_1$  for Monday is consistent and strategy  $S_2$  for Wednesday is consistent. Then there exists a probability measure  $P$  representing both  $S_1$  and  $S_2$  as a single case of Bayesian inference.

**Proof.** Use Theorem 1 to find  $P_1$  representing  $S_1$  and  $P_2$  representing  $S_2$ . Let  $P = P_1 \times P_2$ .

Data = extremely unlikely catastrophic event

# Irreparable error

You cannot add an axiom relating Monday actions and Wednesday actions because that would eliminate some pairs of self-consistent strategies  $(S_1, S_2)$ . Theorem 2 shows that you cannot do that because every pair of self-consistent strategies is a Bayesian strategy.

# Consequences of de Finetti's error

- For Bayesian statistics: none. Admissibility?
- For environment: horrendous.

Bayesian inference is an excellent method of determining exact or approximate values of objective probabilities.



# Frequency philosophy

The von Mises theory can be represented as two scientific ideas. Both were totally rejected.

# Collectives

**Definition:** A collective is a sequence  $X_k$  of 0-1 random variables such that for some  $p$  and every strictly increasing sequence of predictable stopping times  $T_n$

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n X_{T_k} = p, \quad a.s.$$

**Challenge:** Prove CLT for collectives.

I.I.D.  $\implies$  Collective  $\not\Rightarrow$  I.I.D.





# Collectives and only collectives

Von Mises: “Probability theory can be applied only to collectives”.

What should we do with the data on financial markets, climate and weather, social networks, etc.?

## Science of probability (KB)

(L1) Probabilities are numbers between 0 and 1, assigned to events whose outcome may be unknown.

(L2) If events  $A$  and  $B$  cannot happen at the same time then  $P(A \cup B) = P(A) + P(B)$

(L3) If events  $A$  and  $B$  are physically independent then  $P(A \cap B) = P(A)P(B)$

(L4) If there exists a symmetry on the space of possible outcomes which maps an event  $A$  onto an event  $B$  then  $P(A) = P(B)$

(L5) An event has probability 1 if and only if it must occur.

# Popper's view of science

(L5) An event has probability 1 if and only if it must occur.

(L5) is Popper's idea of falsification of probability statements (and scientific statements in general), repackaged for the mass market.

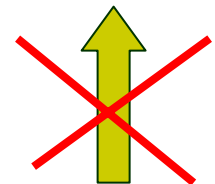
# There is no future

Decisions (actions):

“buy stocks on Monday” and

“buy bonds on Wednesday” cannot be ordered (compared).

	Monday	Tuesday	Wednesday
Physical time	1	2	3
Probabilistic time	$\mathcal{F}$		$\mathcal{G}$



“buy bonds on Wednesday”



De Finetti:

“Our point of view remains in all cases the same: to show that there are rather profound psychological reasons which make the exact or approximate agreement that is observed between the opinions of different individuals very natural, but there are no reasons, rational, positive, or metaphysical, that can give this fact any meaning beyond that of a simple agreement of subjective opinions.”