

This course will focus on the study of classical and multilinear harmonic analysis. In particular, the main goal of the class will be to build the theory of wave packet analysis developed by Carleson and used to solve problems related to convergence of Fourier series and elliptic boundary value problems.

We will follow the texts of Muscalu and Schlag ([MS13a], [MS13b]) as well as the text of Thiele ([Thi06]). If time permits, we will end with more recent methods of decoupling due to Bourgain and Demeter [BD15].

Course Topics:

- Basic Fourier Analysis
 - Fourier Series on \mathbb{T} and Fourier Transform on \mathbb{R} .
 - * Basic definitions and functional setting.
 - Kernels: Dirichlet, Fejer, de la Vallee Poussin
 - * L^p convergence, $1 < p < \infty$
 - * C^α convergence
 - * L^1 Convergence for Fejer and de la Vallee Poussin
 - Bernstein, Sobolev Embedding
 - Weak- L^1 bound for Dirichlet Kernel
 - * Hilbert Transform
 - * Calderon-Zygmund kernel
 - * Interpolation
 - Pointwise Convergence
 - * Convergence for Fejer, de la Vallee Poussin
 - * Counterexample for $f \in L^1(\mathbb{T})$
 - * Statement of Carleson-Hunt Theorem
- Intermediate Theorems
 - Littlewood-Paley
 - * Sub-Gaussian Estimate
 - * Khintchine Inequality
 - General Calderon-Zygmund Theory
 - Stationary and Non-stationary phase
- Wave Packet Decomposition
 - Orthogonal wave packet family construction
 - Dyadic H^1 , dyadic BMO, H^1 -BMO duality
 - Paraproducts
 - $T(1)$ Theorem
 - L^2 -boundedness of C-Z kernel on Lip graph
 - Carleson-Hunt Theorem

References

- [BD15] Jean Bourgain and Ciprian Demeter. The proof of the 1 2 decoupling conjecture. *Annals of mathematics*, pages 351–389, 2015.
- [MS13a] Camil Muscalu and Wilhelm Schlag. *Classical and Multilinear Harmonic Analysis: Volume 1*, volume 137. Cambridge University Press, 2013.
- [MS13b] Camil Muscalu and Wilhelm Schlag. *Classical and Multilinear Harmonic Analysis: Volume 2*, volume 138. Cambridge University Press, 2013.
- [Thi06] Christoph Thiele. *Wave packet analysis*. Number 105. American Mathematical Soc., 2006.