The Impact of Check Sequencing on NSF (Not-Sufficient Funds) Fees

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In a wave of litigation that has escalated into several class-action lawsuits, banks have been charged with sequencing checks unfairly to obtain higher fees for not-sufficient funds (NSF). The point of contention is the banks’ use of the high-low check-sequencing procedure, which presents checks for payment in descending dollar amounts. While it is likely that sequencing plays an important role in determining NSF fees, no studies rigorously analyze the theoretical and empirical properties of different sequencing policies and their financial impact on banks and customers. At the request of a litigation participant, we undertook such a study. We interwove several OR/MS tools, including distribution fitting, simulation, and integer programming. Our results show that sequencing is only half of the story; the other half concerns the role of overdraft protection. By clarifying the impact of check-sequencing policies, our study should help policy makers, regulators, and courts to arrive at a suitable resolution of the issues surrounding the public-policy debate on check-sequencing.

Key words: financial institutions: banks; programming: integer, applications.

History: This paper was refereed.

Some banks’ practice of sequencing checks in an attempt to obtain higher fees for not-sufficient funds (NSF) has met with considerable opposition from consumers in the form of a series of well-publicized lawsuits (Brooks 1999) and class-action suits (Thomas 2001). The point of contention is the banks’ use of the high-low check-sequencing procedure, in which they present checks for payment against the customer’s account in descending dollar amounts. As Brooks noted in his Wall Street Journal article, six of the nation’s nine biggest consumer banks use high-low sequencing policies in at least some of their branches. Furthermore, we know of eight suits against six major banks in four states over the issue of high-low sequencing.

The customer’s checking account agreement usually provides for service charges and fees if the account does not comply with certain conditions, such as the minimum balance requirement. The agreement also authorizes a service charge when a check written by a customer is presented for payment and the account balance is insufficient to cover the check amount. The checks drawn without sufficient available funds in the account are called NSF checks, and the fees banks charge in such instances are called NSF charges (Vergari and Shue 1986). The banks apply the NSF charges to the account after they have processed all the checks for a given day.

Paying banks (banks on whom the checks are drawn) process checks in a variety of ways (Brooks 1999). Some banks process checks against checking account balances on a first-come-first-serve basis, that is, the random order in which the checks are captured in a check-sorting machine. But with increased computer-processing capability, it has become possible for banks to reorder checks in any sequence before posting them against the customer’s account. For example, some banks reorder checks to create a check number sequence, while others sequence checks in ascending (low-high) or descending (high-low) amounts.
The banks that use high-low sequencing offer two arguments in defense (Thomas 2001). First, the Uniform Commercial Code authorizes banks to post checks in any order in most states; and second, customers want their larger checks paid first because they often carry stiffer penalties if they remain unpaid.

In a recent study, the Federal Reserve Bank (Robertson 2002) estimated that annually banks in the US return about 300 million checks out of a total of about 49.6 billion checks. With the average NSF fee of $20.73 per returned check (Board of Governors of the Federal Reserve System 2002), US banks collect about $6.2 billion per year in total NSF fees. Furthermore, as reported by Thomas (2001), the banks that use high-low sequencing bounce from 12 to 24 percent more checks for lack of sufficient funds than other banks. Thus, by using high-low sequencing, the banking industry stands to obtain as much as $1.5 billion in additional NSF fees per year. Given the magnitude of this opportunity to increase revenue, it is not surprising that NationsBank recently paid $9 million to settle a class-action suit out of court rather than move ahead with the discovery phase of the trial (Thomas 2001). The bank did not agree to change its check-sequencing policy.

While some aspects of check-sequencing are obvious, others are not. For example, it is easy to confirm that low-high sequencing can maximize the number of checks cleared (and minimize the NSF charges). But banks do not necessarily maximize NSF charges by using the high-low method. Instead, high-low is simply a heuristic whose effectiveness in practice is unclear and requires further study.

Analyzing the impact of check-sequencing became the task of one of the authors while working on a project for a large consulting firm, whose name we withhold for legal reasons relating to the pending lawsuits. The other authors subsequently contributed to the research to bring independent academic research to bear on an important subject of public-policy interest. In this research project, we collected empirical data, fitted statistical distributions to the data, simulated potential NSF scenarios, and constructed and solved a series of integer programs. The optimization models we used fall into two general categories defined by the presence or absence of overdraft protection, that is, whether customers are permitted to overdraw their accounts up to a preset limit without having their checks returned or not. The results of the two models are remarkably different, but not necessarily along the lines one might anticipate.

Models and Heuristics for Optimizing NSF Charges

We can characterize the NSF problem mathematically as a collection of debits whose sum exceeds the account’s initial balance. We assume that these debits can be presented against the balance in any order, a situation that applies to checks as well as to certain types of electronic transactions, such as debit card charges. For simplicity, we will refer to all debits that can be sequenced as checks.

The phrase “optimizing NSF charges” admits two extremal analyses: one focuses on minimizing bank-related NSF charges and thus favors the bank’s customers; the other focuses on maximizing these charges and thus favors the bank and its shareholders.

Models with No Overdraft Protection

In the absence of overdraft funds, the following notation and decision variables are sufficient to describe the two problems: the consumer’s problem, in which NSF charges are minimized; and the bank’s problem, in which they are maximized.

\[ c_i = \text{the amount of check } i \ (c_i > 0), \ i = 1, \ldots, n. \]

\[ B = \text{the initial available account balance } (B > 0). \]

\[ x_i = \text{a binary decision variable,} \]

\[ x_i = \begin{cases} 1 & \text{if check } i \text{ is cleared}, \\ 0 & \text{if check } i \text{ is returned NSF.} \end{cases} \]

\( \text{NSF} = \text{the NSF charge per check.} \)

We include the condition \( B > 0 \) to eliminate the uninteresting (but common) case in which the available balance is nonpositive and therefore the bank charges an NSF fee for each and every check regardless of the sequence.

We can cast the optimization problem from a consumer’s viewpoint as a simple binary linear program whose objective is to minimize the NSF charges:

\[ \text{Minimize} \sum_{i=1}^{n} \text{NSF} \cdot (1 - x_i) \quad (1) \]

\[ \text{Subject to} \sum_{i=1}^{n} x_i c_i \leq B, \quad (2) \]

\[ x_i \in [0, 1]. \quad (3) \]

Here, the objective function (1) minimizes the sum of NSF charges, while the constraint (2) ensures that the checks cleared do not exceed the available balance. The solution to this problem is straightforward: present checks from the smallest to the largest (low-high). The reasoning behind this is simple. Assume that the optimal sequence clears \( K \) checks, whose sum is denoted by \( S, (S \leq B) \). Because the sum of the first \( K \) checks in a low-high sequence cannot exceed \( S \), the low-high sequence must (also) be an
To assessing four NSF charges for the four returned checks ($675, $525, $75, and $25) (Table 2). Compared to random sequencing with three NSF charges, low-high sequencing reduces the NSF charges to two, while high-low sequencing increases the NSF charges to four.

Despite the more punitive nature of the high-low method, it does not necessarily maximize the number of NSF charges. If the bank had cleared the checks for $525 and $675 first (in either order), the available balance would have been $0, and it would have assessed five NSF charges (Table 3).

In general, we can find the maximum NSF charges for an individual case by solving the following binary linear program:

Maximize \[ \sum_{i=1}^{n} NSF \cdot (1 - x_i) \] \tag{4}

Subject to \[ \sum_{i=1}^{n} x_i c_i \leq B, \] \tag{5}

\[ B x_j + c_j \geq (B + 0.01) - \sum_{i=1}^{n} c_i x_i, \] \[ j = 1, 2, \ldots, n, \] \tag{6}

where \( x_i, c_i > 0 \), and \( B > 0 \) are defined as before. Here, the objective function (4) maximizes the sum of NSF charges, while the constraint (5) ensures that the sum of cleared checks does not exceed the available balance. The system of constraints (6) helps us to identify NSF checks. For example, \( x_i = 1 \) for all checks that are cleared, and in those cases, the constraint (6) is readily satisfied because of the presence of \( B \) on the left-hand side. On the other hand, \( x_i = 0 \) for all NSF checks, and in these cases, the term \( B x_j \) reduces to zero on the left-hand side, allowing the constraint (6) to ensure that the residual balance after clearing checks is strictly smaller than the amount of every NSF check. Although binary linear programs...
are frequently difficult to solve, we encountered no problems with this formulation on the sizes tested.

The solution to this model effectively partitions checks into two sets: checks that clear and NSF checks. The checks to be cleared \( x_i = 1 \) are processed first in any order. In fact, the specific order in which the bank processes checks marked \( x_i = 1 \) within the set has no effect on NSF charges. It next processes the NSF checks \( x_i = 0 \), again in any order. The optimal sequencing policy derived from this model is not necessarily the high-low procedure, which can be viewed as a heuristic approximation to the optimal policy.

**Models with Overdraft Protection**

When the bank provides overdraft protection, the customer may overdraw his or her account up to a preset overdraft limit. For example, with $200 of overdraft protection, a customer can write checks that the bank will honor until the account balance drops to $–200. The bank that provided us with the empirical data charges an NSF fee for each check it honors in whole or in part using overdraft dollars. This fee is fixed. It is not meant to reflect the interest on the amount of overdraft, nor is it proportional to the amount of the check. Although some banks call this fee an overdraft fee instead of an NSF fee, this distinction is somewhat artificial. In both cases, the customer pays a nearly identical fee (per bad check) in addition to repaying the overdraft dollars used. In 2001, for banks of all sizes, the average NSF fee was $20.73, while the average overdraft fee was $20.42 (Board of Governors of the Federal Reserve System 2002). For large banks, the average NSF and overdraft fees were $24.70 and $25.10 respectively.

Although checks honored with overdraft funds incur NSF charges, they are not returned to the merchant. Because returned checks also incur additional charges from merchants, they cause greater total out-of-pocket expenses for customers. Merchants can collect return charges that exceed banks’ NSF charges because, effectively, merchants have been deceived into involuntarily extending credit (Vergari and Shue 1986). Moreover, when banks return some large checks, such as mortgage, credit-card, or auto-loan payments, the merchant can levy a charge that is a percentage of the amount due, which may be much larger than the NSF fee. Bad checks of this type can also severely damage the customer’s credit rating. Thus, overdraft protection can reduce the charges for returned checks and thus reduce total out-of-pocket expenses for customers.

In the presence of overdraft funds, one must allow for the possibility that a check overdraws the account, resulting in a negative balance. This has no effect on NSF charges for low-high sequencing (which remains the theoretical minimum), but for high-low sequencing, NSF charges are a nondecreasing function of the overdraft offered. In the following example, taken from an actual case against Wells Fargo (Thomas 2001), a customer has an initial account balance of $100 and writes two checks, one for $20 and the other for $120. With no overdraft protection, both high-low and low-high will clear the check for $20 and apply an NSF charge to the $120 check. If the customer has a $200 overdraft limit, and the bank presents the check for $120 first, it honors the check with $20 of overdraft and charges an NSF fee. It then honors the $20 check using an additional overdraft of $20 and charges one more NSF fee. Thus, the overdraft protection has resulted in two NSF charges instead of one.

In the previous example in which the bank received seven checks (Tables 1–3), assume that the bank gave a $300 overdraft limit to the customer and used the high-low sequencing method (Table 4).

After the $900 check clears, the account balance drops to $300. This balance, together with the overdraft limit of $300, is still insufficient to clear the next check for $675. It incurs an NSF fee and is returned. However, the $525 check is honored (with an NSF charge) because it requires only $225 of the $300 available overdraft funds. The account balance is now $–225 and the available overdraft funds are $75, so the bank charges an NSF fee for the checks for $200 and $100 and returns them. But it can honor the check for $75 with the remaining overdraft and charge an NSF fee. The account balance is now $–300, and the overdraft limit is reached. Hence, the bank charges an NSF fee for the remaining $25 check and returns it, bringing the total number of NSF charges to six; the maximum number of possible charges for this example. The bank charges NSF fees for some of the checks it honors and for all those it returns. The sequence of honored checks is important inasmuch as the check honored for $525 must precede the check honored for $200.

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### High-Low with Overdraft Protection of $300

<table>
<thead>
<tr>
<th>Balance Available ($)</th>
<th>Check Amount ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,200</td>
<td>900</td>
</tr>
<tr>
<td>300</td>
<td>675</td>
</tr>
<tr>
<td>300</td>
<td>525</td>
</tr>
<tr>
<td>(225)</td>
<td>200</td>
</tr>
<tr>
<td>(225)</td>
<td>200</td>
</tr>
<tr>
<td>(300)</td>
<td>75</td>
</tr>
<tr>
<td>(300)</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 4: Overdraft protection can increase the NSF and overdraft charges collected by banks using the high-low sequencing policy. With overdraft protection, the bank honors a check if it does not overdraw the account by more than the available overdraft (and therefore does not return it to the payee).
$75 if the bank is to charge an NSF fee for the latter. This subtle distinction in sequencing greatly complicates the model for maximizing NSF charges with overdraft protection (Appendix).

The practical implication of this result is that banks that want to increase NSF charges have an incentive to offer overdraft protection. From the customer’s viewpoint, the silver lining is that the bank will honor more checks and therefore not return them. Because of merchants’ fees for bad checks and their effects on customers’ credit ratings, both banks and customers may be better off when banks use high-low sequencing in conjunction with overdraft protection.

Empirical Analysis

We analyzed four sequencing policies: (1) low-high sequencing, which minimizes NSF charges; (2) “random” sequencing, the order in which the bank receives the checks; (3) high-low sequencing, a heuristic approximation to the NSF-maximizing policy; and (4) the NSF-maximizing sequence, determined by solving the appropriate binary linear programs.

We obtained empirical data during the spring of 1999 from a branch of a large national bank that managed approximately 6,500 accounts. Although the bank was planning to implement flexible overdraft limits, it was using the high-low sequencing procedure with $200 of overdraft protection and a $25 NSF fee. Twenty days of checking-account transaction data from accounts held by individuals and small businesses produced 479 NSF cases. The number of checks presented (per evening) against the accounts ranged from one to 16, and the average amount of each check was $264.53. Of these 479 cases, 177 had a positive initial balance (Appendix).

We used this empirical data to conduct a preliminary computational study on those cases in which sequencing could make a difference (a positive balance and more than one check written). In such cases, legal disputes have arisen. Only 69 cases of the 479 met the two criteria. Therefore, in the vast majority of NSF scenarios (410 out of 479), there is little room for dispute. It is likely that overdraft protection would have some impact on these cases, but our focus was on the effects of sequencing, and we did not pursue this line further.

For the cases in which sequencing could make a difference, we implemented the NSF-maximizing binary linear programs in Excel 2000 and solved them using Frontline System’s Premium Solver. Because overdraft limits vary across banks and even across branches of the same bank, we computed the average number of NSF charges and returned checks for each policy with seven different overdraft levels: $0, $100, $200, $300, $400, $500, and $1,000. However, the small size of our data set compromised the stability and robustness of our results. To complicate matters, we lost the possibility of collecting additional data from the source bank when it became a defendant in a lawsuit. Other banks declined to share their data for fear that our results could aid plaintiffs. This situation is not uncommon in legal environments, where knowledge of an unfavorable finding can itself pose a liability.

To resolve the issues surrounding check-sequencing, we resorted to simulation to build a large set of NSF scenarios that was consistent with the original sample (Appendix). This approach provided a larger data set from which we could derive stable and robust results, and it filled in (and smoothed) sample spaces where the original data was thin or nonexistent. For example, the original sample contained no instances of NSF cases with eight checks and positive initial account balances. In estimating long-run probabilities, it is unreasonable to assume that there can be no instances with eight checks. Our simulated data set of 5,000 NSF cases contained 45 cases with eight checks and positive initial account balances. Moreover, the results of this simulation resoundingly confirmed those based on the observed data.

Number of NSF Charges and Returned NSF Checks

For the 5,000 simulated scenarios (all with positive balances and at least two checks written), we computed the average number of NSF charges and the average number of returned checks for the four policies and seven overdraft limits (Table 5). We assumed in our analysis that the customer’s check-writing behavior is not significantly influenced by the amount of overdraft and that customers passed most NSF checks inadvertently due to lapses in bookkeeping. We could not test this assumption thoroughly on our data. However, in 311 of the 479 observed cases, check totals exceeded the overdraft limit available at the source bank ($200), suggesting that inaccurate bookkeeping was indeed a major issue. For these 311 cases, total check amounts exceeded total balances by an average of $791.16.

For a given level of overdraft protection, the average number of NSF charges increases monotonically as one moves from low-high to random, to high-low, and finally to maximize-NSF sequencing. Similarly, for a given sequencing policy, the average number of NSF charges increases monotonically with the level of overdraft protection. What is perhaps surprising is the similarity between the four policies in the absence of overdraft. If we take random sequencing as a point for comparison (this amounts to a bank policy of noninterference), then the low-high sequence reduces charges by only about 1.1 percent, whereas high-low and max NSF increase charges by only about 3.5
and 3.7 percent, respectively. As overdraft increases, the situation changes dramatically. With only $200 of overdraft, high-low produces 42 percent more NSF charges compared to random sequencing without overdraft.

As expected, the low-high sequencing policy produces the smallest number of NSF charges and is therefore the most favorable policy from a customer’s viewpoint. By comparison, the high-low heuristic is extremely efficient at collecting the maximum number of NSF checks. The computational results showed that with $200 of overdraft protection, high-low sequencing led to NSF charges that were identical to those given by the maximize-NSF policy in all but six of 5,000 cases. Considering the total numbers of NSF charges collected for all 5,000 cases, high-low and maximize-NSF sequencing generated 11,607 and 11,616 NSF charges respectively. Thus, high-low sequencing collected 99.92 percent of the maximum possible NSF charges. In short, the high-low sequencing procedure excels in two dimensions essential for any heuristic procedure: simplicity and effectiveness.

Analyzing returned checks required some additional work because the number of returned checks is not well defined for the NSF-maximizing policy with overdraft (Table 5). For example, for an account with an initial balance of $400, overdraft protection of $200, and checks in the amounts of $500, $75, $50, and $35, the maximum number of NSF charges is four, which the bank can achieve with any sequence that presents the check for $500 first. But the number of returned checks varies: it could be two (using the high-low sequence) or it could be only one (presenting the checks in the order $500, $50, $35, and $75). Consequently, we specified a secondary objective to ensure a theoretically consistent solution.

Because maximizing NSF charges is intrinsically a bank-oriented objective, a reasonable secondary objective is minimizing exposure, that is, minimizing the average amount a bank lends to its customer in overdraft funds.

As can be expected, for a given sequencing policy, the average number of returned checks decreases as overdraft protection increases. For a given level of overdraft protection, low-high sequencing produces the smallest and high-low sequencing produces the largest average number of returned checks.

### Table 5: The top portion of the table shows that for a given sequencing policy, the average number of NSF charges increases with the level of overdraft protection, while for a given level of overdraft protection, the average number of NSF charges increases as one moves from low-high, to random, to high-low and finally, to maximize-NSF sequencing. The bottom portion of the table indicates that for a given sequencing policy, the average number of returned checks decreases as the overdraft protection increases. Furthermore, for a given level of overdraft protection, low-high sequencing produces the smallest and the high-low sequencing produces the largest average number of returned checks.

<table>
<thead>
<tr>
<th>Sequencing Policy</th>
<th>None</th>
<th>$100</th>
<th>$200</th>
<th>$300</th>
<th>$400</th>
<th>$500</th>
<th>$1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Number of NSF Charges</td>
<td>1.619</td>
<td>1.619</td>
<td>1.619</td>
<td>1.619</td>
<td>1.619</td>
<td>1.619</td>
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<tr>
<td>None</td>
<td>1.637</td>
<td>1.876</td>
<td>1.967</td>
<td>2.015</td>
<td>2.052</td>
<td>2.077</td>
<td>2.149</td>
</tr>
<tr>
<td>Random</td>
<td>1.694</td>
<td>2.186</td>
<td>2.321</td>
<td>2.399</td>
<td>2.457</td>
<td>2.501</td>
<td>2.607</td>
</tr>
<tr>
<td>High-low</td>
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<td>2.187</td>
<td>2.323</td>
<td>2.400</td>
<td>2.457</td>
<td>2.501</td>
<td>2.608</td>
</tr>
<tr>
<td>Maximize-NSF</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### Financial Impact of Sequencing Policies on Banks and Customers

We used the average number of NSF charges (Table 5) in combination with an assumed average NSF fee of $25 per check to compute the average NSF charge per simulated scenario (Table 6). Again, sequencing had little impact on NSF charges when there was no overdraft. For example, random sequencing with $100 of overdraft is actually more profitable for the bank ($46.90 in NSF fees) than high-low sequencing without overdraft ($42.35 in NSF fees).
To calculate the customer’s total out-of-pocket expense (TOPE), one must consider the NSF fees the bank charged and the other expenses stemming from returned checks. The other expenses related to the returned checks are difficult to estimate accurately because they include merchant-imposed penalties, loss of goodwill, and damage to one’s credit rating. We can, however, develop a reasonable estimate of these expenses. Because customers choose to sign up for the service banks offer, we can reasonably assume that the fee banks charge to honor a check with overdraft funds must be comparable to (and presumably less than) the average penalty the customer incurs for a returned check. We know that the average overdraft fee is nearly identical to the average NSF charge (Board of Governors of the Federal Reserve System 2002). Hence, for simplicity, we assume that the penalty incurred for a returned check is the same as the NSF fee the bank charges, which is approximately $25 on average. We can then estimate the customer’s average TOPE for a given sequencing policy and overdraft level by adding the average number of NSF charges to the average number of returned checks and multiplying the sum by $25 (Table 6). Customers’ resistance to high-low sequencing is understandable given that the average TOPE under high-low sequencing is always higher than that under random sequencing (for the same overdraft limit). However, for a given sequencing policy, the customer’s average TOPE decreases monotonically with the increasing level of overdraft protection. Thus overdraft limits provide benefits for the customers as well as the banks. We cannot resolve the debate over sequencing without including the issue of overdraft limits.

Conclusions
The impact of check-sequencing on NSF charges is a practical issue that has produced considerable rancor and litigation. Empirically, we established that high-low sequencing effectively maximizes NSF charges, but that the impact of this policy is insignificant without some form of overdraft limit. The issuance of overdraft limits benefits both parties: banks increase their NSF charges, and customers reduce their total out-of-pocket expenses by reducing merchant-imposed penalties on returned checks.

We found that sequencing was not an issue in approximately 85 percent of NSF cases because they involved either a single check or a negative initial account balance. Because sequencing cannot increase NSF charges in these situations—even though overdraft limits would still reduce the average number of returned checks—we believe that both parties could reach a satisfactory agreement if overdraft protection were included in the current debate.

Our findings have some limitations that we cannot resolve without more data. For example, we have not modeled the potential relationship between check-writing behavior and overdraft limits. Fortunately, we can resolve these issues as data becomes available. The same OR/MS principles we used in this study will play a central role in understanding the impact of check-sequencing and overdraft limits on NSF fees.

Table 6: The table shows that for a given sequencing policy, the average NSF fees collected by the bank increases as the overdraft level increases while the average total out-of-pocket expenses (TOPE) for the customer decreases as the overdraft level increases.

<table>
<thead>
<tr>
<th>Sequencing Policy</th>
<th>None</th>
<th>$100</th>
<th>$200</th>
<th>$300</th>
<th>$400</th>
<th>$500</th>
<th>$1,000</th>
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</thead>
<tbody>
<tr>
<td>Average NSF Fees per Case</td>
<td>40.47</td>
<td>40.47</td>
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<td>Maximize-NSF</td>
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<td>66.43</td>
<td>61.66</td>
<td>58.80</td>
<td>56.65</td>
<td>55.20</td>
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<td>Average Total Out-of-Pocket Expenses (TOPE) for the Customer</td>
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<td>66.43</td>
<td>61.66</td>
<td>58.80</td>
<td>56.65</td>
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<td>79.14</td>
<td>78.55</td>
<td>77.98</td>
<td>75.36</td>
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</table>

Appendix: Model for Check Processing with Overdraft
We define the following binary decision variables:

\[ x_i = \begin{cases} 1 & \text{if check } i \text{ is cleared (without overdraft funds)} \\ 0 & \text{otherwise} \end{cases} \]

\[ y_i = \begin{cases} 1 & \text{if check } i \text{ is used to overdraw the account} \\ 0 & \text{otherwise} \end{cases} \]

\[ z_i = \begin{cases} 1 & \text{if check } i \text{ is honored with overdraft funds} \\ 0 & \text{otherwise} \end{cases} \]
\[ w_i = \begin{cases} 1 & \text{if check } i \text{ is returned for NSF,} \\ 0 & \text{otherwise.} \end{cases} \]

The variables \( x_i \) and \( w_i \) are self-explanatory, but \( y_i \) and \( z_i \) require some clarification. We need the variable \( y_i \) in some cases to designate the particular check that overdraws the account (creates a negative account balance). We need this variable when other checks honored with overdraft (designated by \( z_i = 1 \)) are NSF only because the account has been overdrawn first. For example, given checks \( c_1 = 20 \), \( c_2 = 120 \), and balance \( B = 100 \) with \( 200 \) of overdraft protection, the bank must present the check for \( 120 \) first to overdraw the account so that it can also charge an NSF fee for the check for \$20\). In our model, we represent this sequence by \( y_1 = 1 \), \( z_1 = 1 \).

We continue to assume that \( c_i > 0 \) and \( B > 0 \). In addition, let \( O \) denote the available overdraft funds. The model for maximizing NSF charges can be expressed as

\[
\text{Max}_{(x, z, y, w)} \sum_{i=1}^{n} \text{NSF} \cdot y_i + \sum_{i=1}^{n} \text{NSF} \cdot z_i + \sum_{i=1}^{n} \text{NSF} \cdot w_i \quad (7)
\]

subject to

\[
\begin{align*}
\sum_{i=1}^{n} c_i x_i & \leq B, \quad (8) \\
\sum_{i=1}^{n} c_i x_i + \sum_{i=1}^{n} c_i y_i + \sum_{i=1}^{n} c_i z_i & \leq B + O, \quad (9) \\
B(1 - y_j) + c_j & \geq (B + 0.01) - \sum_{i=1}^{n} c_i x_i, \\
& \quad (j = 1, \ldots, n), \quad (10) \\
B(1 - z_j) + c_j & \geq (B + 0.01) - \sum_{i=1}^{n} c_i x_i - \sum_{i=1}^{n} c_i y_i, \\
& \quad (j = 1, \ldots, n), \quad (11) \\
(B + O)(1 - w_j) + c_j & \geq (B + O + 0.01) - \sum_{i=1}^{n} c_i x_i - \sum_{i=1}^{n} c_i y_i - \sum_{i=1}^{n} c_i z_i, \\
& \quad (j = 1, \ldots, n), \quad (12) \\
x_j + y_j + z_j + w_j & = 1, \\
& \quad (j = 1, \ldots, n), \quad (13) \\
\sum_{i=1}^{n} y_i & \leq 1, \quad (14) \\
x_i, y_i, z_i, w_i & \in [0, 1]. \quad (15)
\end{align*}
\]

Constraint (8) ensures that the total amount for checks cleared without overdraft protection \((x_i = 1)\) does not exceed the available balance, while constraint (9) ensures that the total amount for the honored checks \((y_i \text{ or } z_i = 1)\) does not exceed the overdraft limit. Constraints (10) guarantee that a check designated by \( y_i = 1 \) is the first to overdraw the account. Constraints (11) ensure that any other check honored with overdraft funds \((z_i = 1)\) exceeds the residual balance by at least one penny. It is possible for \( y_i = 0 \) \( \forall i \), in which case all checks honored with overdraft funds \((z_i = 1)\) must exceed the (nonnegative) residual balance. Constraints (12) are needed to ensure that each check that is returned NSF exceeds the overdraft limit. Constraints (13) ensure that each check receives a unique designation \((x, y, z, \text{ or } w)\).

Constraint (14) is used in conjunction with (10) so that at most one check is labeled the first to overdraw the account. The prevalence of binary variables allows US to solve this model in reasonable time in Excel 2000, with Frontline System’s Premium Solver, for problems containing 12 or fewer checks.

**Data Simulation**

We started with observed data on 479 NSF cases. Our purpose was to extract as much information as possible from these cases when fitting an appropriate distribution for simulating a larger data set. In some cases, only a portion of the observed data was relevant for this purpose.

The first phase of the simulation was to determine the distribution that best fit the number of checks written. Based on the QQ plot, we found that a Pareto distribution with scale parameter 1 and shape parameter 2.2 best fit the 479 observed cases. We chose this Pareto distribution to generate the number of checks, first by generating a Pareto random variable and then taking the integer portion. Truncating the random variable in this fashion provided a better fit than rounding when compared to the empirical distribution of observed cases. We compared the simulated distribution to the empirical distribution for cases where sequencing could potentially make a difference—cases with two or more checks written (Figure 1). There were 190 observed cases with two or more checks written.

We then examined the initial account balances for the 177 observed cases that had a positive initial account balance. Because the initial account balance was related to the number of checks in the observed data, we fit separate distributions depending on the number of checks written. We had enough data to fit five distributions, one for each of the following check-writing scenarios: one check, two checks, three checks, four checks, and five or more checks. We performed the Shapiro-Wilk (1965) test for normality on the natural log of the initial balance for each of these five cases. We could not reject the lognormal fit at the 0.10 level for all five cases. Therefore, we generated the initial balances using these five lognormal distributions.
We then generated the check amounts using simulated ratios of the account balance to the check amount. Once we had the initial balance, we could convert each simulated ratio into an equivalent simulated check amount. When we sorted the observed checks into descending order and divided them into their respective observed balances, we found that we could approximate the second, third, fourth, and fifth ratios (also in descending order) using a lognormal distribution fitted to the observed data. These all passed the Shapiro-Wilk test at the 0.10 level. The first ratio was approximately normal following the square root transformation, although it did not quite pass the Shapiro-Wilk test. Its failure to pass did not necessarily indicate a “bad” fit given that the number of observed cases ($n = 177$) was large enough to distinguish fairly minor departures from normality. In cases with more than five checks, we simulated the amounts for check number six and higher by generating a random number between 0.01 and 1 and then multiplying it by the previous check amount.

We generated 50,000 simulated cases using this methodology. We discarded all cases that were not NSF situations and in which only one check was written because sequencing had no effect in these cases. We eliminated cases in which the starting balance was less than $1 or more than $10,000 because the smallest and largest observed balances in the empirical data set were approximately $2 and $10,000. We also eliminated checks under $1 because such checks are rarely written in practice and were not observed in our original sample. Finally, we discarded cases with more than 12 checks because they made up a negligible percentage of the overall cases (less than 0.3 percent) but added exponentially to our solution times. From the 50,000 simulated cases, after applying the exclusions mentioned above, we took a simple random sample of 5,000 cases for our analysis.

We performed the statistical analysis and simulation using Stata Statistical Software 5.0 for Windows.

**Acknowledgments**

We thank Tony Santomero and Jim Smith for their helpful comments and suggestions on earlier versions of this work.

**References**


We have on file a letter from the lead project consultant for this case at a major US consulting firm attesting to the value of the work described in this paper: “That banks sequence checks to assess NSF fees has become well documented in the popular press (e.g., the *Wall Street Journal*, among others). I am aware of at least six “major” banks that have been sued over this issue.

“The enclosed paper represents an analysis of this practical problem using academic modeling techniques. Although it is difficult to place a dollar figure on this sort of analysis, the results are meaningful to those of us who have been involved with this issue as part of our normal work routine.”