Math 582: Foundations of Combinatorics – Graph Theory
Lecturer: Prof. Sara Billey

Problem Set #8

due Friday, March 9, 2018

Reading: Read Chapter 4 and 5 of Diestel. Plus, "Chip-Firing and Rotor-Routing on Directed Graphs" by Alexander Holroyd, Lionel Levine, Karola Mészáros, Yuval Peres, James Propp and David Wilson. Progress in Probability, Vol. 60, 331364, 2008.

Homework Problems: For each of the problems below, explain your answer fully. No credit will be given for a simple numerical answer.

(1) Prove that every 3-connected graph with at least six vertices that contains a subdivision of K_5 also contains a subdivision of $K_{3,3}$.

(2) Show that adding a new edge to a maximal planar graph of order at least 6 always produces both a K^5 and a $K_{3,3}$ topological minor.

(3) A convex embedding of a graph is one in which every inner face is bounded by a convex polygon. Show that if G is 3-connected and contains no K^5 and a $K_{3,3}$ subdivision then G has a convex embedding.

(4) Does every planar graph have a drawing with all inner faces convex?

(5) A graph is called *outerplanar* if it has a drawing in which every vertex lies on the boundary of the outer face. Show that a graph is outerplanar if and only if it contains neither K^4 nor $K_{2,3}$ as a minor.

(6) Determine the edge chromatic number of the complete graph K^n.

(7) Prove that every maximal planar graphs with chromatic number 3 is Eulerian. (Bonus) Can the reverse implication be made somehow?

(8) (Bonus) Does every graph with chromatic number 4 contain a subdivision of K_4 in which each of the six paths replacing the edges of K_4 has odd length?