Math 582: Foundations of Combinatorics – Graph Theory
Lecturer: Prof. Sara Billey

# Problem Set #7

**due Friday, February 28, 2018**

**Reading:** Read Chapter 3 of Diestel.

**Homework Problems:** For each of the problems below, explain your answer fully. No credit will be given for a simple numerical answer.

(1) Draw the 3-mer graph on the following reads and use it to find ALL shortest possible circular DNA sequence containing all of the reads.

(G C C A C G C) (G A G T A T T) (C C A C G C T) (A T T T T C G)
(T A T T T T C) (G C T T C G A) (A C G C T T C) (C G C T T C G)

(2) Prove that every 3-connected graph with at least six vertices that contains a subdivision of $K_5$ also contains a subdivision of $K_{3,3}$.

(3) Prove that a graph is 2-connected if and only if for every two vertices $x$ and $y$ and every edge $e$ there exists a path from $x$ to $y$ through $e$.

(4) Prove that if $G$ is $k$-connected ($k \geq 2$) then every set of $k$ vertices is contained in a cycle. Is the converse true?

(5) Diestel, Chapter 3, Problem 15.

(6) Diestel, Chapter 3, Problem 18.

(7) (Bonus) Show that any 3-connected planar graph forms the skeleton of a convex polyhedron.