Math 582: Foundations of Combinatorics – Graph Theory
Lecturer: Prof. Sara Billey

Problem Set #5

due Friday, February 8, 2018

Reading: Review "Simplicial Matrix-Tree Theorems" by Art Duval, Carly Klivans, and Jeremy Martin. Also, start reading "How to apply de Bruijn graphs to genome assembly" by Phillip Compeau, Pavel Pevzner and Glenn Tesler.

Homework Problems: For each of the problems below, explain your answer fully. No credit will be given for a simple numerical answer.

(1) Show that the zero-dimensional spanning trees of a simplicial complex are precisely the subcomplexes consisting of a single vertex.

(2) Given a board $B \subset [n] \times [n]$, define a graph G_B such that the rook polynomial of B agrees with the matching polynomial of G_B.

(3) Find an infinite counterexample to the statement of the Hall's Marriage Theorem.

(4) Construct a bipartite graph G with preferences such that some stable matching of the subgraph spanned by $U_1 \cup U_2$ is not a stable matching in G, where $U_1 \cup U_2$ are the matched vertices in every stable matching.

(5) Diestel, Chapter 2, Problem 4.

(6) Consider the simplicial complex Δ with facets $\{123, 124, 134, 234, 125, 135, 235\}$. Show that Δ has 5 0-SST's, 75 1-SST's, and 15 2-SST's. Hint from Lei: for the 2-SST's, use the fact that spheres have non-vanishing top reduced homology. So the goal is to break up the two spheres in $|\Delta|$, just like popping two bubbles.

(7) (Bonus): Construct a family of stable marriages based on n students and n hospitals with a large number of stable marriages. The number of points assigned will be a function of the number of stable marriages for the family.