Math 582: Foundations of Combinatorics – Graph Theory

Lecturer: Prof. Sara Billey

Problem Set #4 due Wednesday, January 30, 2018

Reading: Some topics we are covering this week are not in the textbook. Consult Wikipedia or the other recommended texts as needed. Our first paper will be presented next Wednesday. It is called "Simplicial Matrix-Tree Theorems" by Art Duval, Carly Klivans, and Jeremy Martin. Please read it.

Homework Problems: For each of the problems below, explain your answer fully. No credit will be given for a simple numerical answer.

- (1) Let G_n be the de Bruijn graph with $V = \{(x_1, \ldots, x_n) : x_i \in \{0, 1\}\}$ and directed edges from $(x_1, \ldots, x_n) \leftarrow (x_2, \ldots, x_n, y)$ for both y = 0, 1. Prove the following algorithm produces an Eulerian cycle: Start at vertex $(0, 0, \ldots, 0)$ and follow the edge labeled 1 unless that edge has been used, then follow the edge labeled 0. Continue until return to $(0, 0, \ldots, 0)$ and there are no edges left unused incident to $(0, 0, \ldots, 0)$.
- (2) Do card trick worksheets.
- (3) Invent a new card trick.
- (4) (Bonus): What is the position of a particular binary sequence $b_1b_2\cdots b_n$ in the algorithm for an Eulerian cycle in G_n ?