Math 582: Foundations of Combinatorics – Graph Theory
Lecturer: Prof. Sara Billey

Problem Set #3

due Wednesday, January 24, 2018

Reading: Some topics we are covering this week are not in the textbook. Consult Wikipedia or the other recommended texts as needed. Our first paper will be handed out this week. Please save some time to read it too.

Homework Problems: For each of the problems below, explain your answer fully. No credit will be given for a simple numerical answer.

(1) Use the Matrix Tree Theorem to prove Cayley's Formula.

(2) Use Breadth-First Search to compute the girth of a graph.

(3) Let $K_{3,m}$ be the complete bipartite graph with 3 vertices on the left and m vertices on the right. Compute the number of spanning trees of this graph as a function of m.

(4) Generalize Problem 3 to other complete bipartite graphs.

(5) Let $n \geq r \geq 0$, $E = \{1, 2, \ldots, n\}$, and $\mathcal{B} = \{X \subset E : \ |X| = r\}$. Show $(E, \mathcal{B})$ is a matroid.

(6) Let P be a path in an Eulerian graph G. Prove that G has an Eulerian circuit in which the edges of P appear consecutively (in same order as P) if and only if $G - E(P)$ has only one nontrivial connected component.

(7) Let G_n be the de Bruijn graph with $V = \{(x_1, \ldots, x_n : x_i \in \{0, 1\}\}$ and directed edges from $(x_1, \ldots, x_n) \leftarrow (x_2, \ldots, x_n, y)$ for both $y = 0, 1$. Prove the following algorithm produces an Eulerian cycle: Start at vertex $(0, 0, \ldots, 0)$ and follow the edge labeled 1 unless that edge has been used, then follow the edge labeled 0.